PRECISE & RAPID UNSTEADY PRESSURE TRANSDUCER SIGNAL PROCESSING USING A TRANSFER FUNCTION MODELING TECHNIQUE

Presentation Outline:

- **Introduction**
- **System Identification**
- **Transfer Function Modeling**
- **Difficulties**
- **Conclusions**

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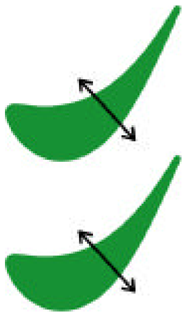
Introduction

BRITE-EURAM Aeromechanical Design of Turbine Blades project (ADTurB I)

CONTROLLED VIBRATION MEASUREMENTS



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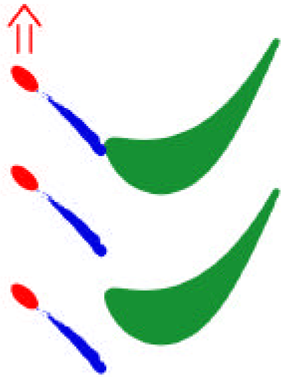
GUST RESPONSE MEASUREMENTS



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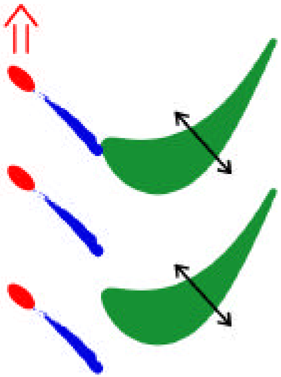
SUPERPOSITION OF CV & GR MEASUREMENTS



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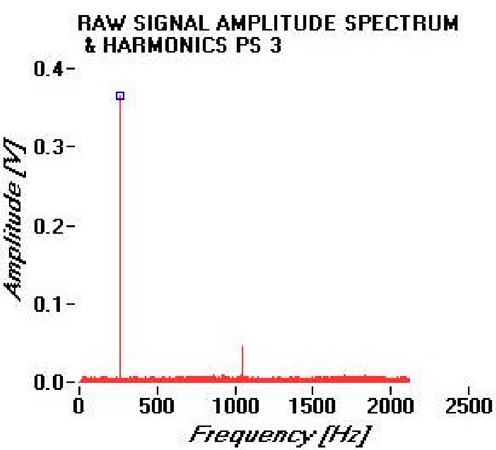
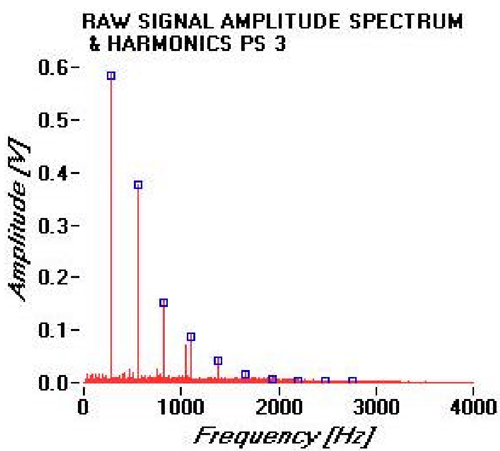
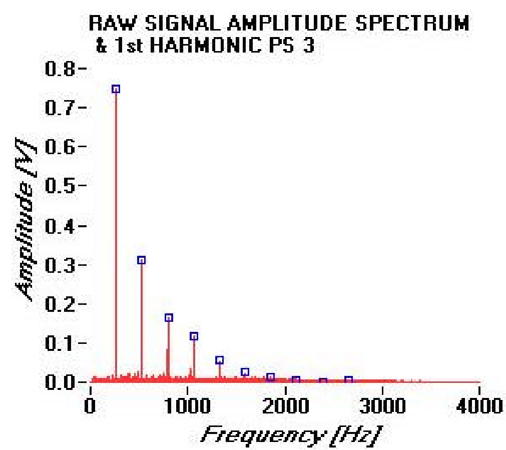
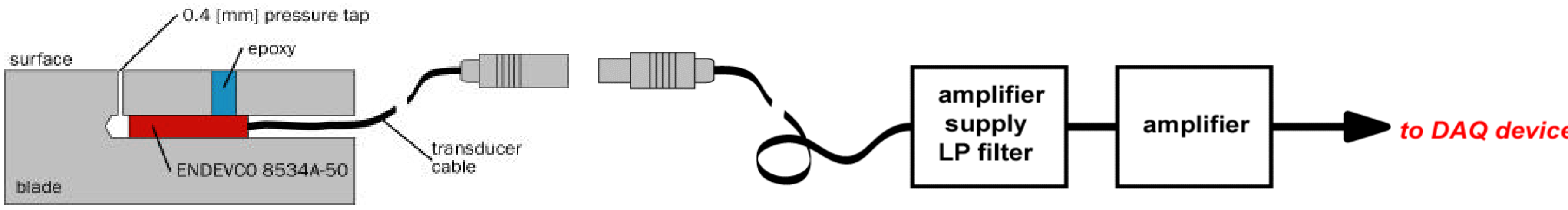


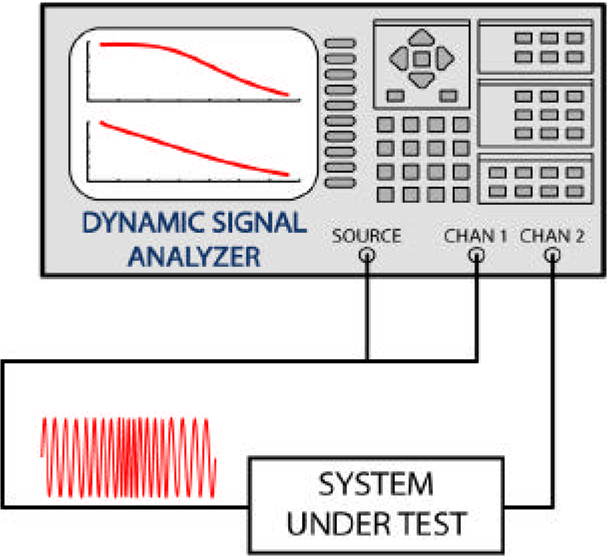


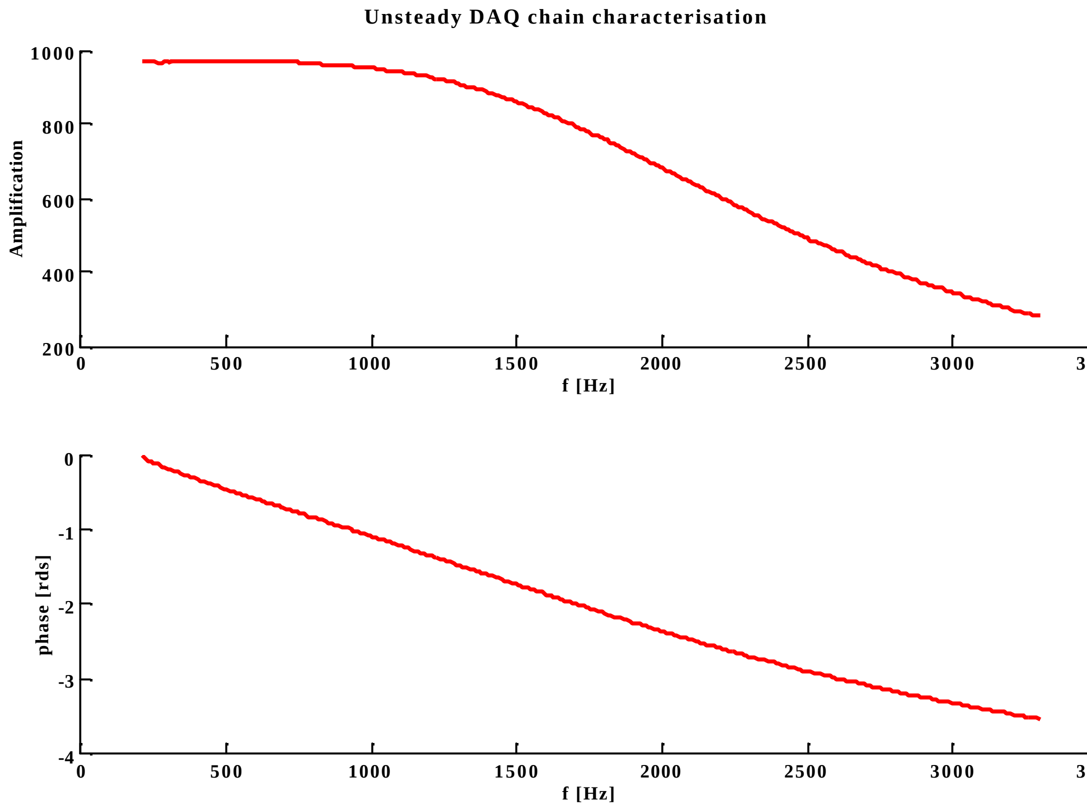


System Identification



Periodic Chirp Response





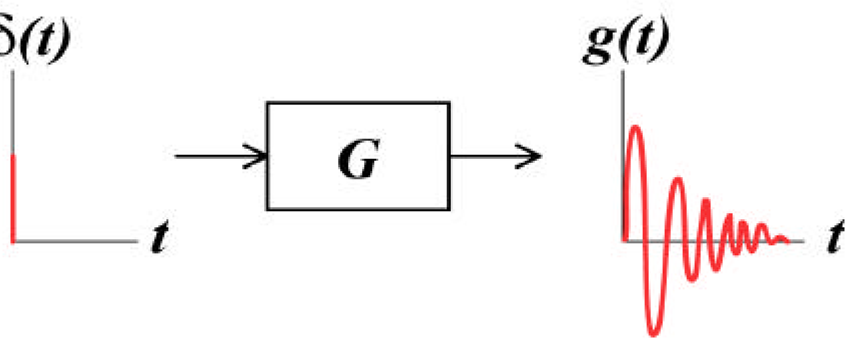
Transfer Function Modeling

<u>H:</u> Linear System

$$y(t) = g(t) * u(t) = \int u(t) \cdot g(t - t)\, dt$$

g(t) **: impulse response**

$$\mathcal{L}[y(t)] = \mathcal{L}[u(t)] \cdot \mathcal{L}[g(t)]$$



transfer function : $G(s) = \mathcal{L}[g(t)]$

$$Y(s) = U(s) \cdot G(s) \quad \Rightarrow \quad \boxed{G(s) = \frac{Y(s)}{U(s)}}$$

(A

Model (2;3) $$G(s) = \frac{\tilde{a}s^2 + \tilde{b}s + \tilde{c}}{s^3 + \tilde{d}s^2 + \tilde{e}s + \tilde{f}}$$ $\tilde{a}, \tilde{b}, \tilde{c}, \tilde{d}, \tilde{e}$ & $\tilde{f} \in$

<u>H:</u> harmonic inputs, transient behavior neglected

$$G(jw) = \frac{\tilde{a}(jw)^2 + \tilde{b}(jw) + \tilde{c}}{(jw)^3 + \tilde{d}(jw)^2 + \tilde{e}(jw) + \tilde{f}} = \frac{\tilde{a} + \frac{\tilde{b}}{jw} + \frac{\tilde{c}}{(jw)^2}}{jw + \tilde{d} + \frac{\tilde{e}}{jw} + \frac{\tilde{f}}{(jw)^2}}$$ **with** $w = 2p$

$$G(jw) = \frac{a + \frac{b}{jw} + \frac{c}{(jw)^2}}{1 + d \cdot jw + \frac{e}{jw} + \frac{f}{(jw)^2}}$$ **(B)** **where**

$$a = \frac{\tilde{a}}{\tilde{d}}; \quad b = \frac{\tilde{b}}{\tilde{d}}; \quad c = \frac{\tilde{c}}{\tilde{d}};$$

$$d = \frac{1}{\tilde{d}}; \quad e = \frac{\tilde{e}}{\tilde{d}}; \quad f = \frac{\tilde{f}}{\tilde{d}}$$

(A) & (B) yield:

$$Y(jw) = -d\cdot(jw)\cdot Y(jw) - \frac{e}{jw}\cdot Y(jw) - \frac{f}{(jw)^2}\cdot Y(jw) + a\cdot U(jw) + \frac{b}{jw}\cdot U(jw) + \frac{c}{(jw)^2}\cdot U(jw)$$

Introducing $\widehat{Y}$: the estimated *Y* value

$$\widehat{Y}(jw) = -d\cdot(jw)\cdot Y(jw) - \frac{e}{jw}\cdot Y(jw) - \frac{f}{(jw)^2}\cdot Y(jw) + a\cdot U(jw) + \frac{b}{jw}\cdot U(jw) + \frac{c}{(jw)^2}\cdot U(jw)$$

Collecting the n measured samples:

n=387

$$\boldsymbol{Y} = \begin{bmatrix} Y(jw_1) \\ \vdots \\ Y(jw_i) \\ \vdots \\ Y(jw_n) \end{bmatrix} \qquad \boldsymbol{U} = \begin{bmatrix} U(jw_1) \\ \vdots \\ U(jw_i) \\ \vdots \\ U(jw_n) \end{bmatrix}$$

$$\boxed{\widehat{\boldsymbol{Y}} = \boldsymbol{R}\cdot J} \quad \textbf{(C)}$$

Regression matrix *R*:

$$\boldsymbol{R} = \begin{bmatrix} \boldsymbol{U} & \frac{\boldsymbol{U}}{jw_i} & \frac{\boldsymbol{U}}{(jw_i)^2} & -\boldsymbol{Y}\cdot jw_i & -\frac{\boldsymbol{Y}}{jw_i} & -\frac{\boldsymbol{Y}}{(jw_i)^2} \end{bmatrix}$$

Vector of parameters J :

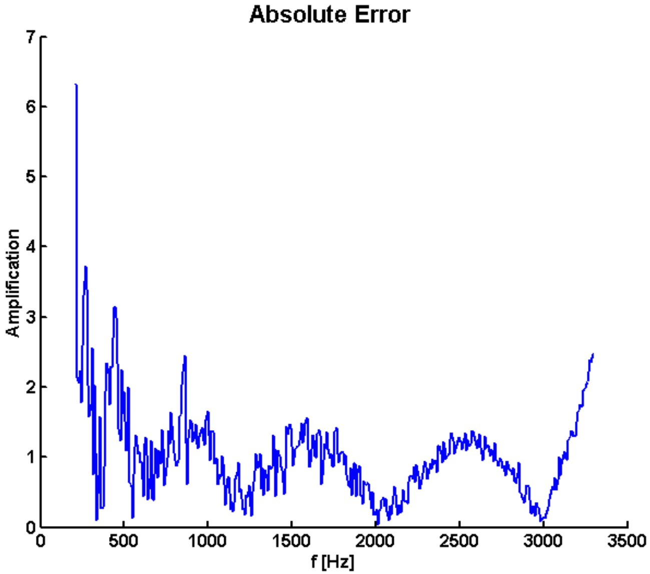
$$J = \begin{bmatrix} a & b & c & d & e & f \end{bmatrix}^T$$

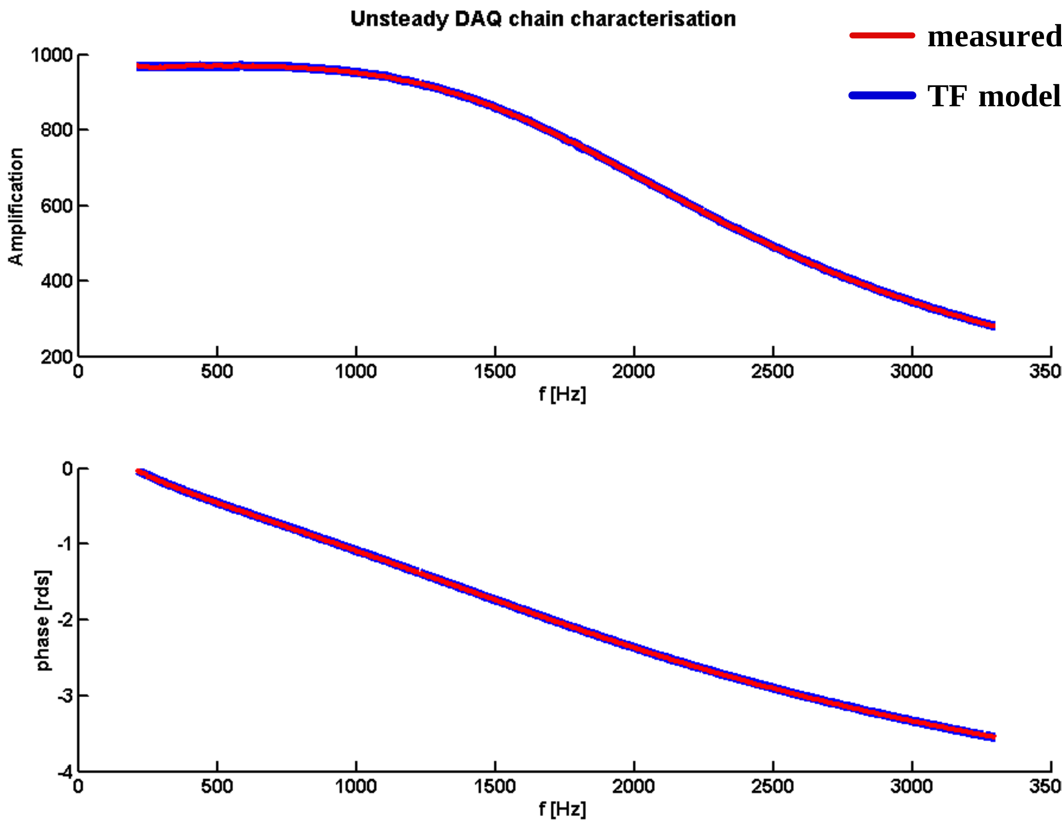
Result

Optimal parameter vector based on a least square criteria:

$$J^* = \arg\min_J \left(\mathbf{Y} - \widehat{\mathbf{Y}}(J)\right)^T \left(\mathbf{Y} - \widehat{\mathbf{Y}}(J)\right) \Rightarrow J^* = \left(\mathbf{R}^T\mathbf{R}\right)^{-1}\mathbf{R}^T\mathbf{Y}$$

Absolute error: $e = \left|\mathbf{Y} - \widehat{\mathbf{Y}}\right|$





Difficulties

- $\boldsymbol{R}^T\boldsymbol{R}$ **is close to singular, parameters can be inaccurately computed during inversio**

- **If** $\boldsymbol{R}^T\boldsymbol{R}$ **is not invertible or** J^* **is inaccurate, one can introduce a scaling matrix** $\mathring{a}$

$$J^* = \arg\min_J \left(\boldsymbol{Y} - \widehat{\boldsymbol{Y}}(J)\right)^T \mathring{a} \left(\boldsymbol{Y} - \widehat{\boldsymbol{Y}}(J)\right)$$

$$J^* = \left(\boldsymbol{R}^T \mathring{a}\, \boldsymbol{R}\right)^{-1} \boldsymbol{R}^T \mathring{a}\, \boldsymbol{Y}$$

- **At low frequencies, the phase angles can be negative violating the linearity assumptio**

Solution: reject these undesired values

Conclusions

- **Successful use of Transfer Modeling Technique to Unsteady Pressure Transducer Signal Processing**
- **The Transfer Modeling Technique is more precise than previously used curve fitting method (error 5 times smaller in this example)**
- **Corrections are easily implemented: multiplication of the digitized signal by the inverse of the Transfer Function Model yields the corrected signal.**
- **The Transfer Modeling Technique allows much faster signal processing; especially if one considers the full pressure signal (harmonics + noise)**
- **The Transfer Function Model and the corresponding parameters estimation algorithm can be directly applied to similar cases.**

Questions

definitions:

```
Ampl = Bmat;
Phas = Amat*pi/180;
omeg = 2*pi*Afreq;
Uomeg = ones(size(Ampl));
Yomeg = Ampl.*exp(j*Phas);
Q = diag(1);
divis = (j*omeg).^2;
```

algorithm:

```
R = [Uomeg, Uomeg./(j*omeg), Uomeg./divis,- Yomeg.*(j*omeg), -Yomeg./(j*omeg), -Yomeg./divis];
Yreg = Yomeg;
ParaEstim = inv(R'*Q*R)*R'*Q*Yreg;
reconstr = R*ParaEstim;
err = reconstr - Yreg;
```

plot:

```
figure(1);
plot(Afreq,abs(err));
figure(2);
subplot(211);
plot(Afreq, abs(Yomeg), Afreq, abs(reconstr))
subplot(212);
plot(Afreq, unwrap(angle(Yomeg)), Afreq, unwrap(angle(reconstr)));
```

xport estimators:

```
format long E;
A = real(ParaEstim);
B = imag(ParaEstim);
C = [A,B];
dlmwrite('..\UPT_TF_estimators.txt',C,'\t');
```