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REYNOLDS STRESS MEASUREMENT WITH 1D LDA

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Outline

- Motivation for technique
  - Capital cost
  - Equipment availability
  - Processing technique to extend measurements possible for 1D LDA
- Derive Equations
  - Mean flow and Turbulent statistics
  - Improve mean and turbulent data using least squares
- Demonstrate Technique
  - Back to back comparison of rotated 1D and 2D LDA
  - Show effect of number of angles and number of samples on measurement quality

Reynolds Stresses

- Reynolds stress (2D)

$$\overline{uv} = \tfrac{1}{N}\sum_{i=1}^{N}\frac{(U-\overline{U})(V-\overline{V})}{UV}$$

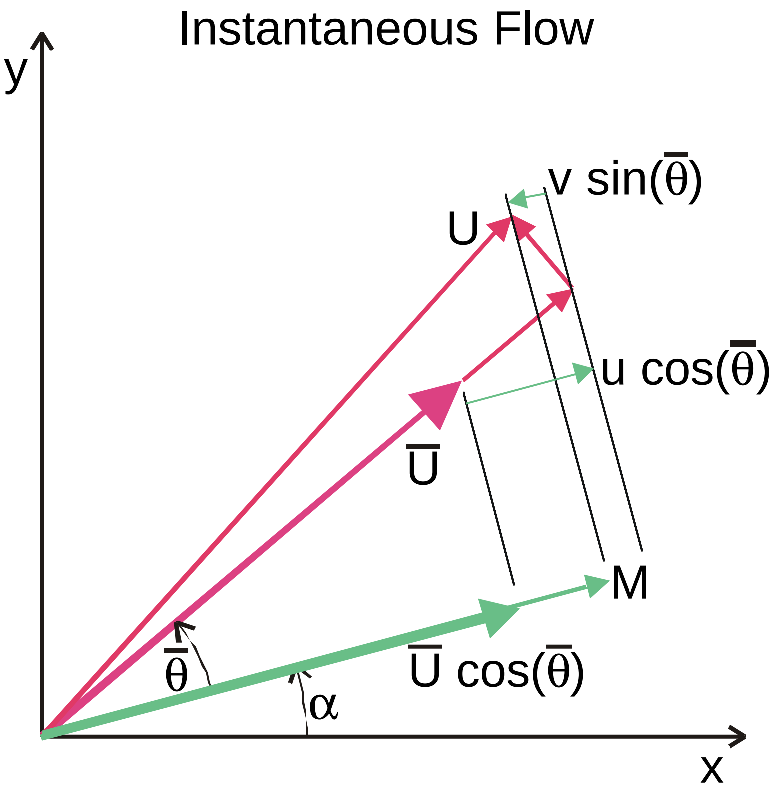
- Require two simultaneous components of velocity
- Calculate covariance
- For steady or phase locked flows we can make the measurements independently and recombine them to get the covariance

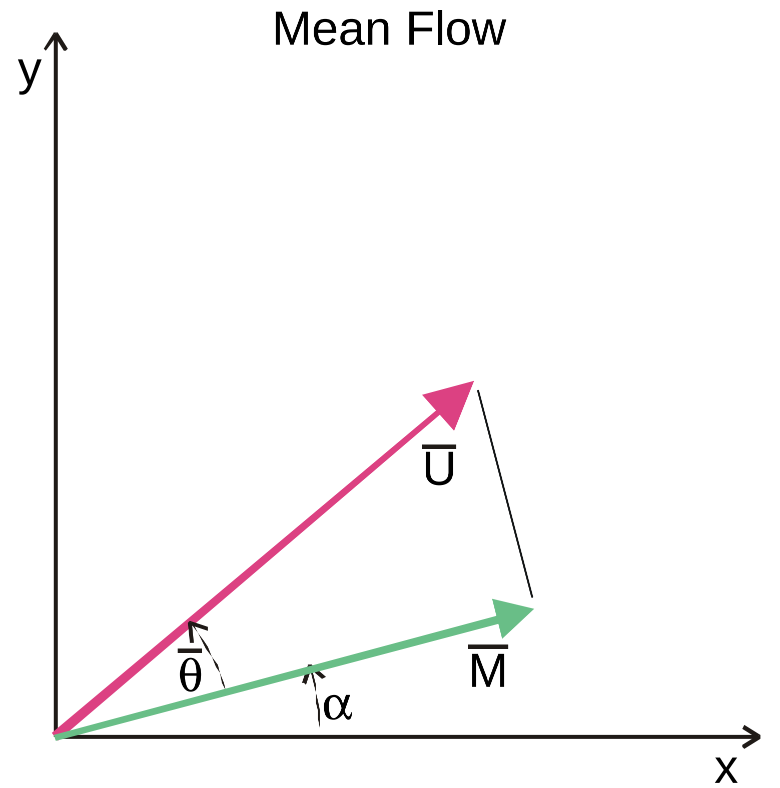
Basis of Technique

- LDA measures velocity component in the plane of the intersecting beams
- Pure cosine relationship between measured velocity and probe angle relative to flow

$$M = U\cos(q)$$

Derivation





$M=(\overline{U}+u)\cos(\overline{\theta}) - v\ \sin(\overline{\theta})$

$\overline{M}=\overline{U}\cos(\overline{\theta})$

subtract mean from each sample, then square, then average

$$\overline{m^2} = \overline{u^2}\cos^2\overline{q} + \overline{v^2}\sin^2\overline{q} - 2\overline{uv}\sin\overline{q}\cos\overline{q}$$

If $\overline{m^2}$ and $\overline{q}$ known then 3 unknowns

thus 3 probe orientations required

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Find the mean Flow

- Fit measured data onto known angular response of LDA
- Minimise error S if more than 2 probe angles used
  - Non-linear in variables $\overline{U}$ and $\overline{q}$
  - Use numerical approach
    - eg Levenberg-Marquardt algorithm

$$S = \sum_{i=1}^{N} \left(\overline{M}_i - \overline{U} \cos(\overline{q}_i) \right)^2$$

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Find turbulent flow

- Functional relationship is

$$\overline{m^2} = \overline{u^2}\cos^2\overline{q} + \overline{v^2}\sin^2\overline{q} - 2\overline{uv}\sin\overline{q}\cos\overline{q}$$

- know $\overline{q}$ (previous slide) and $\overline{m^2}$ (measurement) from more than 3 probe orientations
- Use least squares fit (linear) to improve data

$$\sum_{i=1}^{N} A(\overline{q}_i)\cdot\begin{bmatrix}\overline{u^2}\\ \overline{v^2}\\ \overline{uv}\end{bmatrix} = \sum_{i=1}^{N}\overline{m_i^2}\begin{bmatrix}\cos^2(\overline{q}_i)\\ \sin^2(\overline{q}_i)\\ \cos(\overline{q}_i)\sin(\overline{q}_i)\end{bmatrix}$$

$$A(\overline{q}_i) = \begin{bmatrix}\cos^4(\overline{q}_i) & \cos^2(\overline{q}_i)\sin^2(\overline{q}_i) & \cos^3(\overline{q}_i)\sin(\overline{q}_i)\\ \cos^2(\overline{q}_i)\sin^2(\overline{q}_i) & \sin^4(\overline{q}_i) & \cos(\overline{q}_i)\sin^3(\overline{q}_i)\\ \cos^3(\overline{q}_i)\sin(\overline{q}_i) & \cos(\overline{q}_i)\sin^3(\overline{q}_i) & \cos^2(\overline{q}_i)\sin^2(\overline{q}_i)\end{bmatrix}$$

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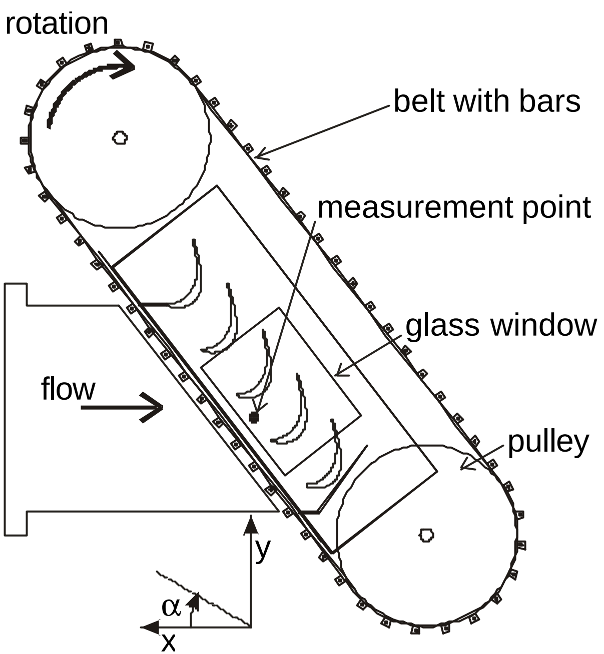
Application of Technique

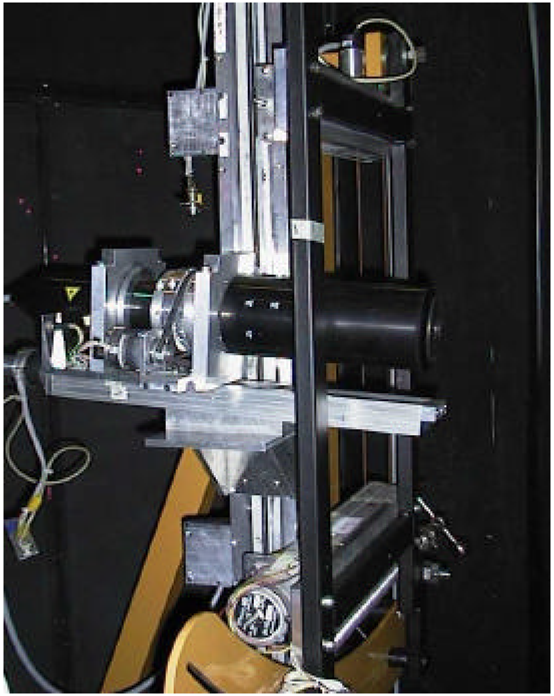
- Measured wake of passing bar
  - simple flow
  - phase locked by trigger
- 120 000 samples at each orientation
- 128 phase bins for ensemble averaging



- 2D measurement at multiple probe orientations, extract 1D data and compare with 2D data

LDA Equipment

- 5W Argon-Ion laser
- 1D used 514nm, 2D also 488nm beam
- Dantec BSA signal processor
- 2D 85mm Probe
- 500 mm focal length
- Stepper motor traverse for probe rotation



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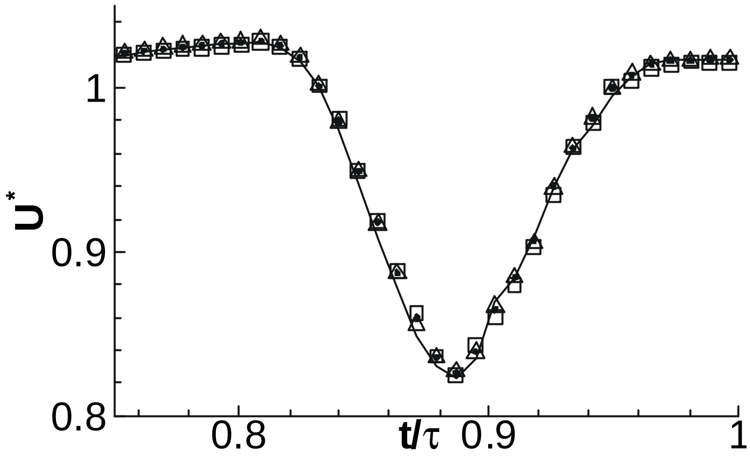
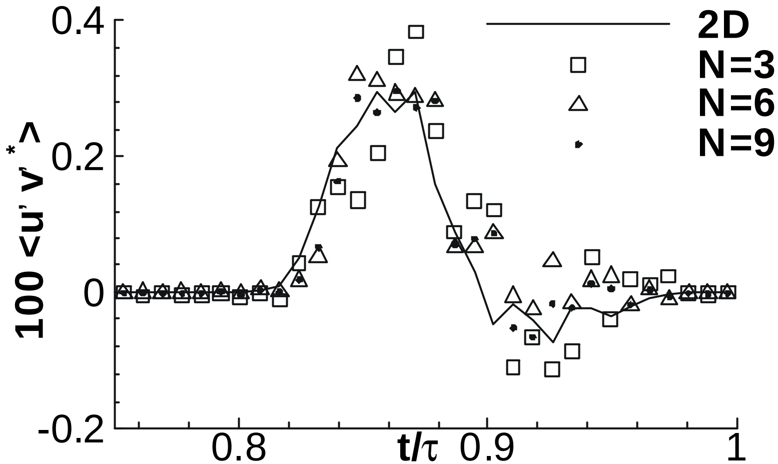
Evaluation of rotated 1D LDA

- mean
  - good agreement for all numbers of angles (N)



- turbulent statistics
  - improve with number of samples (number of angles)



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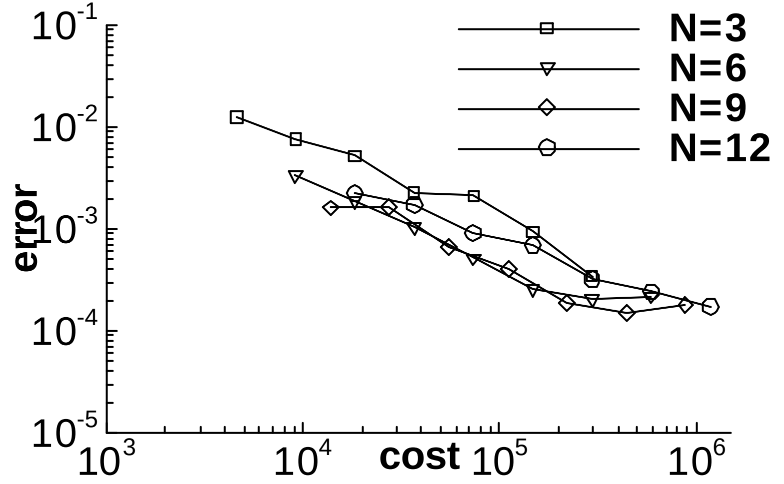
Error vs. Cost

- Estimate error in <uv> as variance of 1D from 2D results
- Cost is total number of samples used



- high error at all costs for N=3
  - shows benefit of least squares approach
- highest N not necessarily lowest error
  - need to have good estimates for mean and variance of each measurement angle

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Conclusion / Discussion

- Exact angular response
  - No need for angular calibration nor small angle assumptions of equivalent HWA technique
- Spatial resolution of LDA maintained
- No directional ambiguity (LDA)
- Reduced capital cost traded against increased acquisition time

Other Application

- dissipation measurement

$$C_D = \frac{1}{rU_\infty{}^3}\int_0^d \frac{1}{T} t_{yx} \frac{du}{dy} dy$$

$$t_{yx} = m\frac{du}{dy} - r\overline{u'v'}$$