

The tangential components of the E field are continuous across the interface

The difference of the tangential components of the H field is equal to the surface current density

The difference of the normal components of D is equal to the surface charge density

The normal components of B are continuous across the interface

In the absence of surface charges or currents, the electric and magnetic fields are continuous.

How to derive them?- Examples $\int_{S} \underline{B} \cdot d\underline{S} = \int_{Uppersurface} \underline{B} \cdot d\underline{S} + \int_{Lowersurface} \underline{B} \cdot d\underline{S} + \int_{Sidesurface} \underline{B} \cdot d\underline{S} = 0$ $\Delta h \to 0 \Rightarrow \int_{Uppersurface} \underline{B}_{1} \cdot \underline{n} dS - \int_{Lowersurface} \underline{B}_{2} \cdot \underline{n} dS + 0 = 0$ M = 0





By definition of η in medium 1 and 2 we have:

$$\eta_1 = \frac{E_{xi}}{H_{yi}} = -\frac{E_{xr}}{H_{yr}}$$

and

$$\eta_2 = \frac{E_{xt}}{H_{yt}}$$

Eliminating $H_{y_{i_1}}H_{y_r}$, H_{y_t} using η_1 , η_2 , gives:				
E_{xi} E_{xr} E_{xt}				
$\frac{-\pi}{\eta_1} - \frac{-\pi}{\eta_1} = \frac{-\pi}{\eta_2}$				
since $E_{xi} + E_{xr} = E_{xt}$	¢			
Eliminating E _{xt} gives:				
$\rho_r = \frac{E_{xr}}{E_{xi}} = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2}$	Reflection Coefficient	(6.1a)		
Eliminating E _{xr} gives:				
$\rho_t = \frac{E_{xt}}{E} = \frac{2\eta_2}{E} T$	ransmission Coefficient	(6.1b)		
E_{xi} $\eta_1 + \eta_2$		150		
Incident power density	$\mathbf{P}_i = \frac{\left E_{xi} \right ^2}{2\eta_1}$			
Incident power density Transmitted power density	$P_{i} = \frac{\left E_{xi}\right ^{2}}{2\eta_{1}}$ $P_{t} = \frac{\left E_{xt}\right ^{2}}{2\eta_{2}}$			
Incident power density Transmitted power density Reflected power density	$P_{i} = \frac{\left E_{xi}\right ^{2}}{2\eta_{1}}$ $P_{t} = \frac{\left E_{xt}\right ^{2}}{2\eta_{2}}$ $P_{r} = \frac{\left E_{xr}\right ^{2}}{2\eta_{1}}$			
Incident power density Transmitted power density Reflected power density $P_{r} = \left(\frac{\eta_{1} - \eta_{2}}{\eta_{1} + \eta_{2}}\right)^{2} P_{i} =$	$P_{i} = \frac{\left E_{xi} \right ^{2}}{2\eta_{1}}$ $P_{t} = \frac{\left E_{xt} \right ^{2}}{2\eta_{2}}$ $P_{r} = \frac{\left E_{xr} \right ^{2}}{2\eta_{1}}$ $= \rho_{r}^{2} P_{i}$			
Incident power density Transmitted power density Reflected power density $P_{r} = \left(\frac{\eta_{1} - \eta_{2}}{\eta_{1} + \eta_{2}}\right)^{2} P_{i} = \frac{4\eta_{1}\eta_{2}}{(\eta_{1} + \eta_{2})^{2}} P_{i} = \frac{4\eta_{1}\eta_{2}}{(\eta_{1} + \eta_{2})^{2}} P_{i}$	$P_{i} = \frac{\left E_{xi} \right ^{2}}{2\eta_{1}}$ $P_{t} = \frac{\left E_{xt} \right ^{2}}{2\eta_{2}}$ $P_{r} = \frac{\left E_{xr} \right ^{2}}{2\eta_{1}}$ $= \rho_{r}^{2} P_{i}$ $(1 - \rho_{r}^{2}) P_{i} \qquad \text{NOT } \rho_{i}$	t ² Pi		



The wave travels in medium 1 from C to B in the same time as it does from A to D in medium 2

Hence
$$\frac{CB}{AD} = \frac{v_1}{v_2}$$

But $CB = AB \sin \theta_i$
 $AD = AB \sin \theta_i$
 $\frac{AD}{AD} = AB \sin \theta_i$
 $\frac{\sin \theta_i}{\sin \theta_i} = \frac{v_1}{v_2}$
Since $v = \frac{1}{\sqrt{\mu \varepsilon}}$
 $\frac{\sin \theta_i}{\sin \theta_i} = \frac{\sqrt{\mu_2 \varepsilon_2}}{\sqrt{\mu_1 \varepsilon_1}} = \frac{n_2}{n_1}$ Snell's Law of Refraction (6.2)
Where n is called refractive index
For dielectrics we assume $\mu_1 = \mu_2 = \mu_0$
This assumption will be used in the rest of the lectures
Then
 $\frac{\sin \theta_i}{\sin \theta_i} = \left(\frac{\varepsilon_2}{\varepsilon_1}\right)^{\frac{1}{2}} = \frac{\eta_1}{\eta_2}$
Do not confuse n with η
Since $\eta = \sqrt{\frac{\mu}{\varepsilon}}$
Do not confuse n with η



By a similar argument it can be shown that the angle of reflection is equal to the angle of incidence

AE=CB
but

$$CB = AB \sin \theta_i$$
 $AE = AB \sin \theta_r$
 $intermodeline in the end of th$





II.3.3 Perpendicularly polarised waves

In these waves the electric field is perpendicular to the plane of incidence i.e. parallel to the boundary between the two media.

Light with perpendicular polarization is called s-polarized (s=senkrecht, perpendicular in German).



From equation 6.3:

$$\frac{E_r^2}{E_i^2} = 1 - \left(\frac{\varepsilon_2}{\varepsilon_1}\right)^{\frac{1}{2}} \frac{E_t^2 \cos \theta_t}{E_i^2 \cos \theta_i} = 1 - k \frac{E_t^2}{E_i^2}$$

Where $k = \left(\frac{\varepsilon_2}{\varepsilon_2}\right)^{\frac{1}{2}} \frac{\cos \theta_t}{\varepsilon_1}$

Where

$$\left(\frac{\boldsymbol{\mathcal{E}}_2}{\boldsymbol{\mathcal{E}}_1}\right)^{\frac{1}{2}} \frac{\cos \boldsymbol{\theta}_t}{\cos \boldsymbol{\theta}_i}$$

But, in this case $E_i + E_r = E_t$

We then get the following:

$$\frac{E_r^2}{E_i^2} = 1 - k \frac{(E_r + E_i)^2}{E_i^2}$$

 $\checkmark \quad \left(\frac{E_r}{E_{\cdot}}\right)^2 (1+k) + \left(\frac{E_r}{E_{\cdot}}\right)(2k) + (k-1) = 0$

This has the same form as

$$ax^2 + bx + c = 0$$

The positive solution is

$$\frac{E_r}{E_i} = \frac{1-k}{1+k}$$

Thus

or

$$\frac{E_r}{E_i} = \frac{n_1 \cos \theta_i - n_2 \cos \theta_t}{n_1 \cos \theta_i + n_2 \cos \theta_t}$$

 $\frac{E_r}{E_i} = \frac{\varepsilon_1^{1/2} \cos \theta_i - \varepsilon_2^{1/2} \cos \theta_t}{\varepsilon_1^{1/2} \cos \theta_i + \varepsilon_2^{1/2} \cos \theta_t}$

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This expression contains the angle of the transmitted wave

We can use Snell's law to obtain a more useful expression which contains only the angle of incidence

From Snell's Law:
$$\frac{\sin \theta_i}{\sin \theta_t} = \left(\frac{\varepsilon_2}{\varepsilon_1}\right)^{1/2}$$

But

$$\cos\theta_t = \sqrt{1 - (\sin\theta_t)^2} = \sqrt{1 - \frac{\varepsilon_1}{\varepsilon_2} (\sin\theta_i)^2}$$

Then

$$\frac{E_r}{E_i} = \frac{\cos\theta_i - \left(\frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i\right)^{1/2}}{\cos\theta_i + \left(\frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i\right)^{1/2}} = \frac{n_1 \cos\theta_i - n_2 \sqrt{1 - \left(\frac{n_1}{n_2} \sin\theta_i\right)^2}}{n_1 \cos\theta_i + n_2 \sqrt{1 - \left(\frac{n_1}{n_2} \sin\theta_i\right)^2}} \quad (6.4)$$





In this case E is no longer parallel to the reflecting plane. The boundary condition applies to the component of E parallel to the reflecting plane:

 $E_i \cos \theta_i - E_r \cos \theta_i = E_t \cos \theta_r$

Following through the algebra our expression for the ratio of reflected to incident waves becomes:

$$\frac{E_r}{E_i} = \frac{\varepsilon_2^{1/2} \cos \theta_i - \varepsilon_1^{1/2} \cos \theta_t}{\varepsilon_2^{1/2} \cos \theta_i + \varepsilon_1^{1/2} \cos \theta_t} = \frac{n_2 \cos \theta_i - n_1 \cos \theta_t}{n_2 \cos \theta_i + n_1 \cos \theta_t}$$
(6.5)

$$\frac{E_r}{E_i} = \frac{\left(\frac{\varepsilon_2}{\varepsilon_1}\right)\cos\theta_i - \left(\frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i\right)^{1/2}}{\left(\frac{\varepsilon_2}{\varepsilon_1}\right)\cos\theta_i + \left(\frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i\right)^{1/2}} = \frac{n_2\cos\theta_i - n_1\sqrt{1 - \left(\frac{n_1}{n_2}\sin\theta_i\right)^2}}{n_2\cos\theta_i + n_1\sqrt{1 - \left(\frac{n_1}{n_2}\sin\theta_i\right)^2}}$$
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Note: normal incidence corresponds to $\theta_i=0$

$$\theta_i = 0 \implies \frac{E_r}{E_i} = \frac{n_2 - n_1}{n_2 + n_1}$$

(6.5) is similar to equation (6.4), but in (6.5) the numerator can become zero, i.e. no reflected wave

The angle at which this occurs is known as the Brewster Angle

Setting the numerator of (6.5) equal to zero we get

Remembering that $\cos \theta_i = (1 - \sin^2 \theta_i)^{1/2}$ $\sin^2 \theta_i = \frac{\tan^2 \theta_i}{1 + \tan^2 \theta_i}$

$$\tan(\theta_B) = \left(\frac{\varepsilon_2}{\varepsilon_1}\right)^{\frac{1}{2}} = \frac{n_2}{n_1} \qquad \theta_B \text{ is called Brewster Angle}$$

Note (6.4) can go to zero only for $n_1=n_2$: i.e. travel in the same medium 167





II.3.6 Total internal reflection The previous section showed graphs for $\frac{\mathcal{E}_2}{\mathcal{E}_2} > 1$. I.e. our wave is moving from a lower refractive index medium to a higher one If instead $\frac{\mathcal{E}_2}{\mathcal{E}_1}$ < 1 then the phenomenon known as total internal reflection can occur The term $\left(\frac{\varepsilon_2}{\varepsilon_1} - \sin^2 \theta_i\right)$ in equations (6.4) or (6.5) can be negative when $\sin \theta_i \ge \left(\frac{\varepsilon_2}{\varepsilon_1}\right)^{n/2} = \frac{n_2}{n_1}$ $\sin \theta_{cr} = \left(\frac{\varepsilon_2}{\varepsilon_1}\right)^{1/2} = \frac{n_2}{n_1}$ The critical angle is defined as 172 For $\theta_i = \theta_{cr}$ $E_r = 1$ For $\theta_i > \theta_{cr}$ the term $\left(\frac{\varepsilon_2}{\varepsilon_i} - \sin^2 \theta_i\right)$ in (6.4) or (6.5) is negative its square root is imaginary and $\frac{E_r}{r}$ is complex E_{\cdot} $\frac{E_r}{E_i} = \frac{A - iB}{A + iB} \qquad \left(\frac{E_r}{E_i}\right)^* = \frac{A + iB}{A - iB}$ $\left|\frac{E_r}{E}\right|^2 = \left(\frac{E_r}{E}\right)\left(\frac{E_r}{E}\right) = 1$ i.e. the magnitudes of the incident and reflected power are equal.

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This gives rise to *total internal reflection*: the incident wave is reflected at the boundary and no wave emerges from the higher refractive index medium

This is shown in the graph below at all angles of incidence where:







II.3.7 Example: reflected power

Diamond has $\varepsilon_r = 5.84 \& \mu_r = 1$. What power fraction of light is reflected off an air/diamond surface for normal incidence?

Recalling that for transmission lines:

$$\overline{\rho}_{L} = \frac{\overline{V}_{B}}{\overline{V}_{F}} = \frac{\overline{Z}_{L} - Z_{0}}{\overline{Z}_{L} + Z_{0}}$$

Similarly for E-M waves, for the case where E and H are parallel to the reflecting plane: $n_{\rm E}$

$$\rho_{r} = \frac{E_{xr}}{E_{xi}} = \frac{\eta_{Diamond} - \eta_{Air}}{\eta_{Diamond} + \eta_{Air}} = \frac{\eta_{Air}}{\eta_{Diamond} + 1} \quad \text{with} \quad \eta = \sqrt{\frac{\mu}{\varepsilon}}$$
Then $\frac{\eta_{Diamond}}{\eta_{Air}} = \frac{\sqrt{\frac{\mu_{r}\mu_{0}}{\varepsilon_{r}\varepsilon_{0}}}}{\sqrt{\frac{\mu_{0}}{\varepsilon_{0}}}} = \sqrt{\frac{\mu_{r}}{\varepsilon_{r}}} = \sqrt{\frac{1}{5.84}} \approx 0.41 \quad \text{min} \left|\overline{\rho}_{R}\right|^{2} = \left|\frac{0.41 - 1}{0.41 + 1}\right|^{2} \approx 17.5 \frac{0}{0}$

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III Antennae and Radio Transmission

Aims

To give a qualitative description of antennae and radio transmission

Objectives

At the end of this section you should be able to recognise the main classes of antennae, and do simple calculations of radiation resistance, effective area, gain.

III.1 Antennae The aim of an antenna is to get signal power from the transmitter to the receiver circuit as efficiently as possible Transmitter Receiver 178

III.1.1 Slot and aperture antennae

Almost any guide carrying an electromagnetic wave will radiate part of the wave if its end is open



The electromagnetic waves in a guide will radiate if you chop its end off (very inefficient)

The ideal antenna sends as much radiation as possible in the desired direction and with the minimum of internal reflection.



Bigger aperture so

-Less diffraction and more gain (directed power, see III.2) -The launched wave is more similar to a plane wave -There is a gradual change between the electrical wave, with characteristic impedance Z_0 , and the radiated wave, with intrinsic impedance η_0 , hence less reflection.

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Horn antennae such as these work very well but they are bulky, therefore unwieldy

1965 Penzias & Wilson used this horn antenna to detect the background microwave radiation of the universe. 1978 Nobel Prize Physics

III.1.1.2 Laser



A laser can be thought of as an aperture antenna and the output roughly approximates to a plane wave.

III.1.2 Dipole antennae

III.1.2.1 Half-wave dipole

If the end of a transmission line is left open circuit, the current at the end is 0 and the reflection coefficient is 1

$$\rho_L = \frac{Z_L - Z_0}{\overline{Z}_L + Z_0}$$

if $Z_L \to \infty$ then $\rho_L \to 1$

In this case, the telegrapher's equations show that $1\!\!\!/_4$ of a wavelength back from the load the voltage must be 0









III.1.2.3 Half dipole

Conductors reflect radio waves because E=0 at the conductor (that is why mirrors are shiny)

A single dipole placed above a conductor radiates like one half of a dipole pair

Long wave radio masts are like this, and can be formed by covering the ground with wire mesh $\hfill \square$



III.1.3 Loop antennae

The loop antenna is like the half-wave dipole except that it behaves like a transmission line whose end is a short circuit and is therefore driven by a voltage source



Short Circuit a $\lambda/4$ transmission line..

If the end of a transmission line is a short circuit, $Z_L=0$



The telegrapher's equations show that 1/4 of a wavelength back from the load the current must be 0.



Inside a portable radio you will find a ferrite rod wound with copper wire. This is the long wave antenna. It has several loops and a ferrite core. The core concentrates the electromagnetic waves into the antenna and the whole arrangement is essentially half of a transformer



III.1.4 Reflector antennae

Another way of concentrating the electromagnetic waves is using a parabolic mirror

Placing a source, S, at the focus of a parabaloid can result in a very directive beam



as the spacings (a,b) and the phases $(e^{j\alpha}, e^{j\beta})$ are correct

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III.2 Radio transmission III.2.1 Radiation resistance

Antennae emit power, so they can be modelled as resistors:

The radiation resistance, R_a , of an antenna is that resistance which in place of the antenna would dissipate as much power as the antenna radiates

$$R_a = \frac{P_{ant}}{(I_{rms})^2}$$
(8.1)

where

$$P_{ant} = \int_{s} k I_{ant}^2 d\underline{S} = k \int_{s} I_{ant}^2 d\underline{S} \qquad (8.2)$$

and
$$kI_{ant}^2 = \frac{1}{2} \operatorname{Re}\left\{\overline{E} \times \overline{H}^*\right\}$$
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The power is calculated by integrating over a surface far from the radiating antenna which encloses the current carrying portion

For example, a sphere at a distance **r** away from the axis of a wire carrying a current I_{ant}

As the distance r increases E varies as E_0/r and H as $E_0/r\eta$

These are termed the far field values of the electromagnetic radiation from the antenna

The intensity of power from a real antenna is not uniform. It has a 3 dimensional pattern, which is termed the radiation pattern

Example: consider a half dipole antenna with:

III.2.2 Gain

The gain, G, of an antenna is the factor by which its maximum radiated intensity exceeds that of an isotropic antenna if they emit equal power from an equal distance

$$G = \frac{\frac{1}{2} \operatorname{Re} \left\{ \overline{\underline{E}}_{ant}(r,\theta,\varphi) \times \overline{\underline{H}}_{ant}^{*}(r,\theta,\phi) \right\}_{Max}}{\frac{1}{2} \operatorname{Re} \left\{ \overline{\underline{E}}_{iso}(r,\theta,\varphi) \times \overline{\underline{H}}_{iso}^{*}(r,\theta,\phi) \right\}} = \frac{kI_{ant}^{2}}{R_{a}I_{iso}^{2}/4\pi r^{2}} \quad (8.3)$$
Provided that
$$\int \underline{Int}_{antenna} \cdot \underline{dS} = \int \underline{Int}_{isotropic} \cdot \underline{dS}$$
An isotropic antenna is a hypothetical device which radiates equally in all directions.

III.2.3 Effective area

The effective area, A_{eff} , of an antenna is that area of wavefront whose power equals that received from the wavefront by the antenna

 $A_{eff} = \frac{\text{Power collected by antenna}}{\text{Wave intensity (i.e. power / area) into antenna}}$

Note: even though the area of radio wave intercepted by a parabolic dish antenna is obvious, it is not in other cases

For example: in principle a half wave dipole could have no area at all and yet still receive power from a radio wave. Hence the need to define an effective area.

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III.2.4 Example: Power transmission

If two half-wave dipoles are 1 km apart and one is driven with 0.5 amps (RMS) at 300 MHz, what power is received by the other ?

G = 1.64,
$$R_a = 73 \ \Omega$$
, $A_{eff} = 0.13 \ m^2$

Intensity r metres from an isotropic antenna

= Transmitted power/ $(4\pi r^2)$

Intensity r metres from this antenna = $G \frac{I_{iso}^2 R_a}{4 \pi r^2}$

Power received by receiving antenna = Intensity $\cdot A_{eff}$

$$= G \frac{I_{iso}^{2} R_{a}}{4\pi r^{2}} A_{eff} = 1.64 \cdot \frac{0.5^{2} \cdot 73}{4\pi (1000)^{2}} \cdot 0.13 = 0.3 \,\mu\text{W}.$$

Optional Examples

O1-Negative refractive index

O2-Invisibility cloak

O3-Hard disk drives

O4-Transparent conductors

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O1-Negative refractive index

In 1967 Viktor Veselago theoretically investigated the properties of media with a negative permittivity ϵ together with negative permeability μ in the same frequency range

He predicted that the wave vector of a wave propagating through such a medium is antiparallel to its Poynting vector



This has far-reaching consequences:

A wave impinging from vacuum onto the surface of such a medium under an angle with respect to the surface normal will be refracted towards the "wrong" side of the normal, i.e., we obtain negative refraction





$$\mathcal{E}(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$
 $\omega_p = \sqrt{\frac{4\pi ne^2}{m}}$ Plasma Frequency

With n electron charge density and m electron mass



However a large magnetic response, in general, and a negative permeability μ at optical frequencies, in particular, do not occur in natural materials **207**

A negative index of refraction can be implemented by an array of metallic split-ring resonators (SRR)



SRR act as *LC*-oscillators. Since the capacitance and inductance are determined by the dimensions of the SRR, scaling of the structure allows to tune the resonance frequency from microwave to terahertz to the infrared

Prerequisite for the magnetic response is the excitation of a circulating current in the individual SRR by the incident field. This induces a magnetic field which can lead to an effective negative permeability μ

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O2-Invisibility cloak



Susumu Tachi 2003 However: Optical Camouflage! Project in front what seen in back. Not "genuine" see through 209

Invisibility cloak (genuine!)

This is another example of a new optical device which could result from our ability to tailor an optical material properties

It relies on a controlled spatial variation of permittivity and permeability guiding light around the central part of the cloak



The plane wave impinging from left is "flowing" around the cloak without being disturbed by the metallic cylinder in the middle 210





This cloaking device is practically invisible...

If you see the world in microwaves with a wavelength of 3.5cm

Schurig et al. Science 2006

Simulations



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O3-Hard disk drives





Parallel Recording



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Perpendicular Recording









Mini-Hard Drive



O4-Transparent conductors

Flexible, Foldable AMOLED Display







- Front Plane : Touch Screen, OLED
- Back plane : TFTs

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Touch screen displays



Electronic paper



Photovoltaic cells



Sensors



Radio frequency tags



Smart textile

ITO (Indium-Tin-Oxide) drawbacks

- Increasing cost due to Indium scarcity
- Processing requirements, difficulties in patterning
- Sensitivity to acidic and basic environments
- Brittleness
- Wear resistance



Bendability of Electronic Materials

Material	Fracture Strain	Material	Fracture Strain
Silicon	~ 0.7%	Poly- ZnO	0.03%
ITO	0.58 ~ 1.15%	Polyimide	4%
Au	0.46%	Graphene	>15-20%







