

## II.3 Reflection and refraction of EM Waves

### Aims

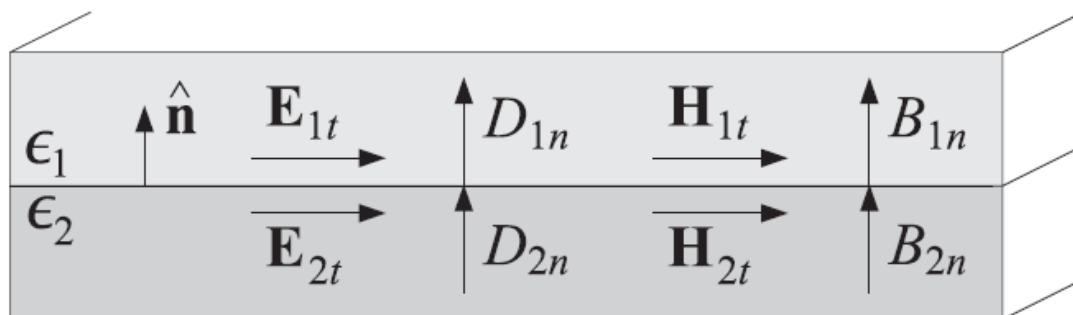
To describe the reflection and refraction of waves from pure dielectric surfaces

### Objectives

At the end of this section you should master the boundary conditions for  $\mathbf{E}$  and  $\mathbf{H}$  fields, be able to calculate the reflection and refraction angles, the reflected and transmitted power and be aware of the Fresnel formulae.

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### II.3.0 Boundary conditions



$$E_{1t} - E_{2t} = 0$$

$$H_{1t} - H_{2t} = \mathbf{J}_s \times \hat{\mathbf{n}}$$

$$D_{1n} - D_{2n} = \rho_s$$

$$B_{1n} - B_{2n} = 0$$

$\mathbf{n}$  is a unit vector normal to the boundary, pointing from medium 2 to medium 1

$\rho_s$  and  $\mathbf{J}_s$  are surface charge and surface current densities on the boundary surface

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The tangential components of the E field are continuous across the interface

The difference of the tangential components of the H field is equal to the surface current density

The difference of the normal components of D is equal to the surface charge density

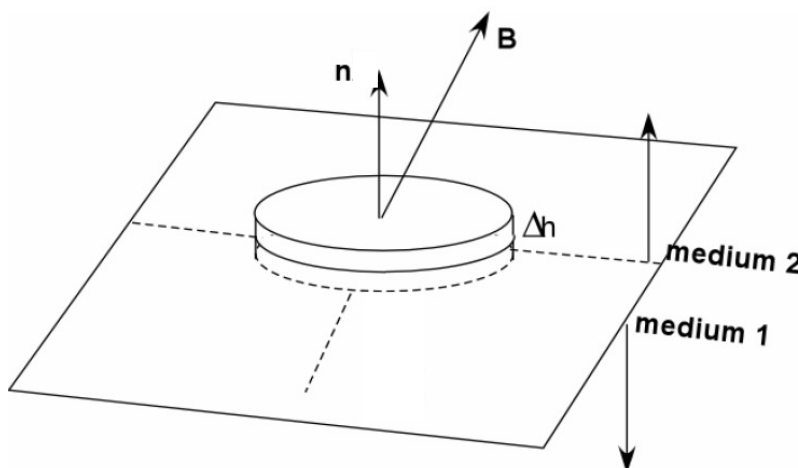
The normal components of B are continuous across the interface



In the absence of surface charges or currents, the electric and magnetic fields are continuous.

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### How to derive them?- Examples



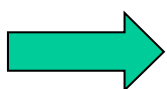
From Gauss' Law

$$\int_S \underline{B} \cdot d\underline{S} = 0$$



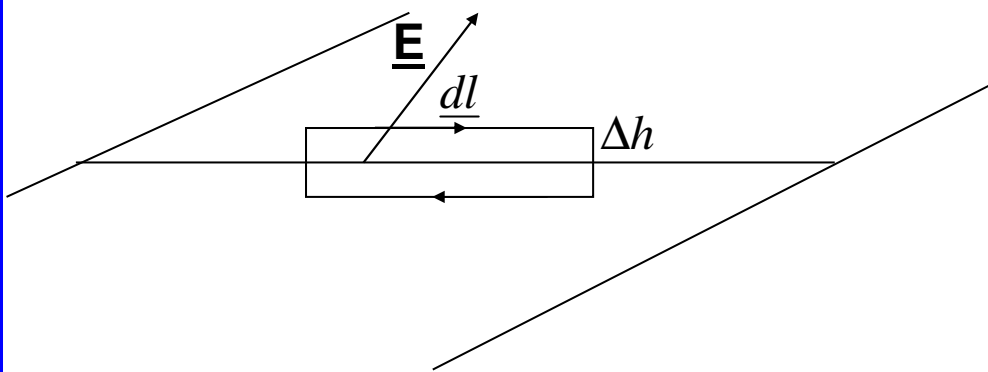
$$\int_S \underline{B} \cdot d\underline{S} = \int_{Uppersurface} \underline{B} \cdot d\underline{S} + \int_{Lowersurface} \underline{B} \cdot d\underline{S} + \int_{Sidesurface} \underline{B} \cdot d\underline{S} = 0$$

$$\Delta h \rightarrow 0 \Rightarrow \int_{Uppersurface} \underline{B}_1 \cdot \underline{n} dS - \int_{Lowersurface} \underline{B}_2 \cdot \underline{n} dS + 0 = 0$$



$$B_{1n} - B_{2n} = 0$$

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From Maxwell-Faraday  $\oint_c \underline{E} \cdot d\underline{l} = - \int_s \underline{\dot{B}} \cdot d\underline{S}$

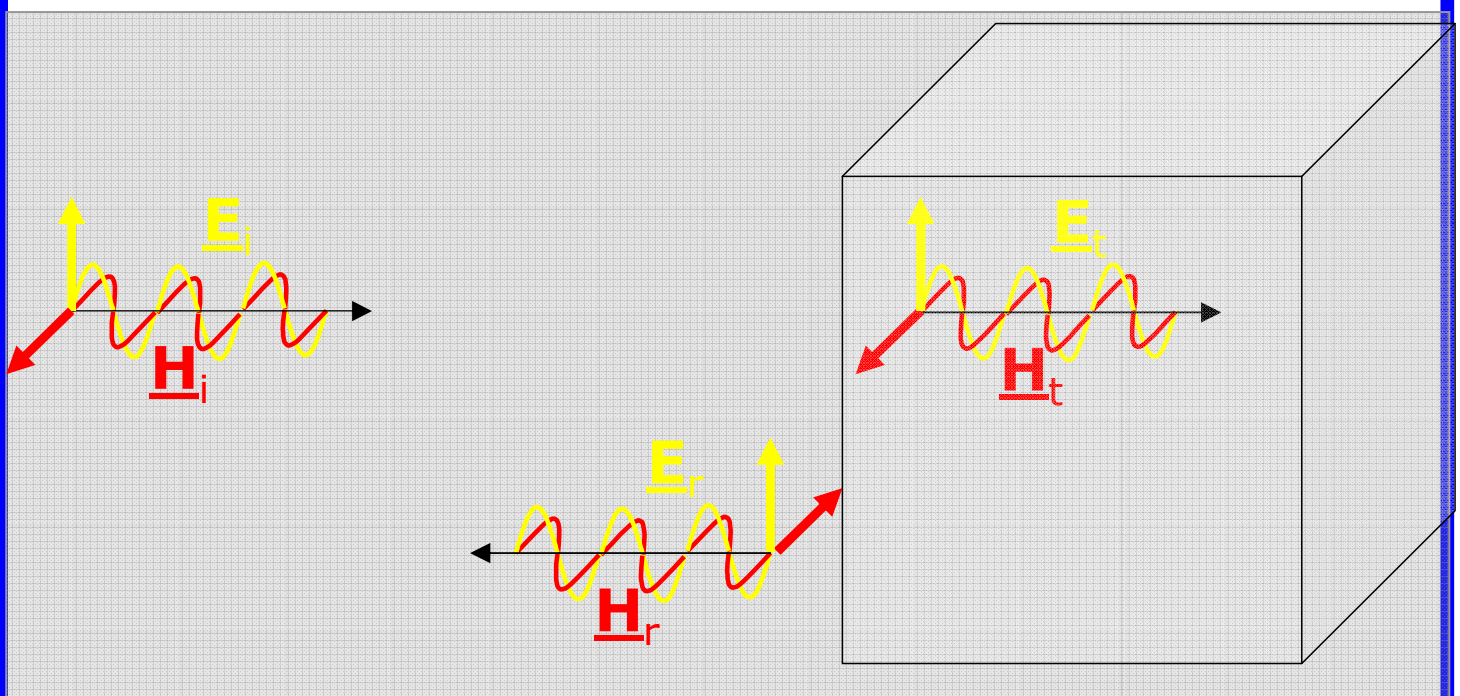
$$\Delta h \rightarrow 0 \quad \Rightarrow \quad S = dl \Delta h \rightarrow 0$$

$$\oint_c \underline{E} \cdot d\underline{l} = \underline{E}_1 \cdot d\underline{l} - \underline{E}_2 \cdot d\underline{l} = 0$$

➡  $E_{1t} - E_{2t} = 0$

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## II.3.1 Reflection of incident wave normal to the plane of reflection



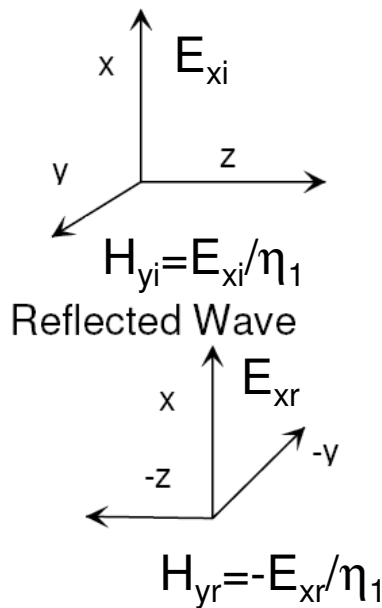
This is a special case, which could be derived by analogy with reflection of  $V$  and  $I$ . We now derive the same results by matching the boundary conditions: 147

## Boundary Conditions

In the absence of surface charges or currents, the electric and magnetic fields are continuous

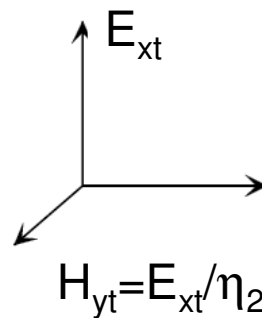
→ the total field in medium 1 is equal to the total field in medium 2

Incident Wave



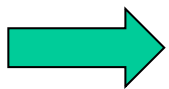
Medium 1

Transmitted Wave



Medium 2

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$$E_{xi} + E_{xr} = E_{xt}$$

$$H_{yi} + H_{yr} = H_{yt}$$

By definition of  $\eta$  in medium 1 and 2 we have:

$$\eta_1 = \frac{E_{xi}}{H_{yi}} = -\frac{E_{xr}}{H_{yr}}$$

and

$$\eta_2 = \frac{E_{xt}}{H_{yt}}$$

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Eliminating  $H_{yi}$ ,  $H_{yr}$ ,  $H_{yt}$  using  $\eta_1$ ,  $\eta_2$ , gives:

$$\frac{E_{xi}}{\eta_1} - \frac{E_{xr}}{\eta_1} = \frac{E_{xt}}{\eta_2}$$

since  $E_{xi} + E_{xr} = E_{xt}$

Eliminating  $E_{xt}$  gives:

$$\rho_r = \frac{E_{xr}}{E_{xi}} = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2} \quad \text{Reflection Coefficient} \quad (6.1a)$$

Eliminating  $E_{xr}$  gives:

$$\rho_t = \frac{E_{xt}}{E_{xi}} = \frac{2\eta_2}{\eta_1 + \eta_2} \quad \text{Transmission Coefficient} \quad (6.1b)$$

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Incident power density  $P_i = \frac{|E_{xi}|^2}{2\eta_1}$

Transmitted power density  $P_t = \frac{|E_{xt}|^2}{2\eta_2}$

Reflected power density  $P_r = \frac{|E_{xr}|^2}{2\eta_1}$

$$P_r = \left( \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2} \right)^2 P_i = \rho_r^2 P_i$$

$$P_t = \frac{4\eta_1\eta_2}{(\eta_1 + \eta_2)^2} P_i = (1 - \rho_r^2) P_i \quad \text{NOT } \rho_t^2 P_i$$

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## II.3.2 Reflection of a wave at oblique incidence on a dielectric boundary

The behaviour of the reflected and transmitted waves will depend on the orientation of  $E$  with respect to the boundary

We therefore split the incident waves into two polarised parts

1) Perpendicularly polarised - electric field at right angles to incident plane

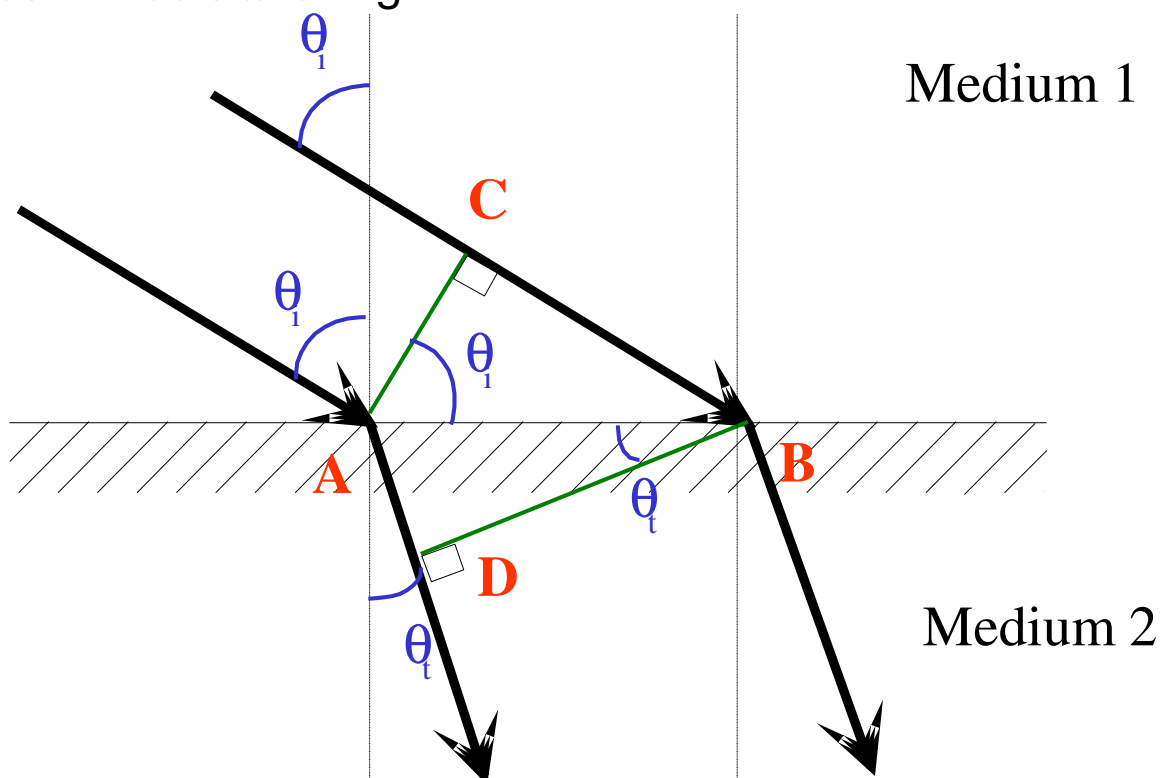
2) Parallel polarised - electric field in incident plane

### Fresnel Formulae (1819)

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#### II.3.2.1 Snell's law of refraction (Snellius 1621/ Ibn Sahl 984)

Before calculating the reflection and refraction coefficients we need to know the angles at which the reflected and refracted waves will be travelling



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The wave travels in medium 1 from C to B in the same time as it does from A to D in medium 2

Hence 
$$\frac{CB}{AD} = \frac{v_1}{v_2}$$

But 
$$CB = AB \sin \theta_i$$
$$AD = AB \sin \theta_t$$

 
$$\frac{\sin \theta_i}{\sin \theta_t} = \frac{v_1}{v_2}$$

Since 
$$v = \frac{1}{\sqrt{\mu \epsilon}}$$

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$$\frac{\sin \theta_i}{\sin \theta_t} = \frac{\sqrt{\mu_2 \epsilon_2}}{\sqrt{\mu_1 \epsilon_1}} = \frac{n_2}{n_1}$$
 Snell's Law of Refraction (6.2)

Where n is called refractive index

For dielectrics we assume  $\mu_1 = \mu_2 = \mu_0$

**This assumption will be used in the rest of the lectures**

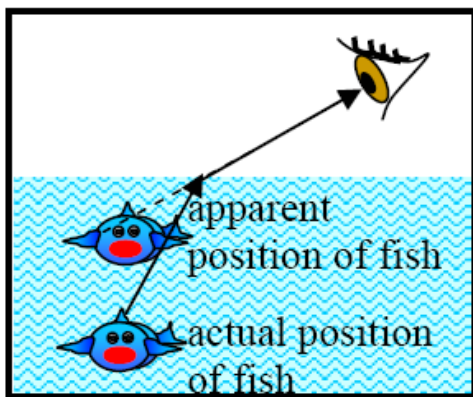
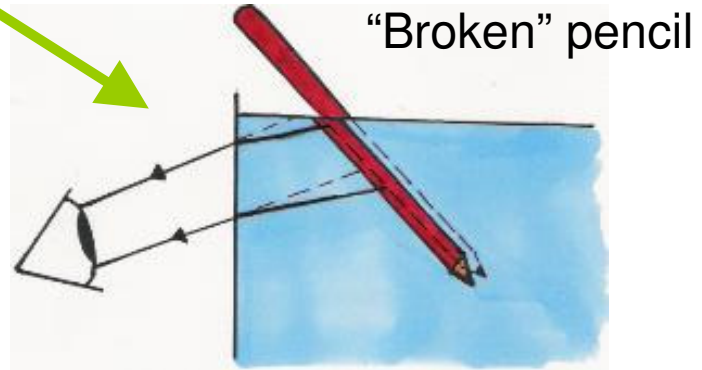
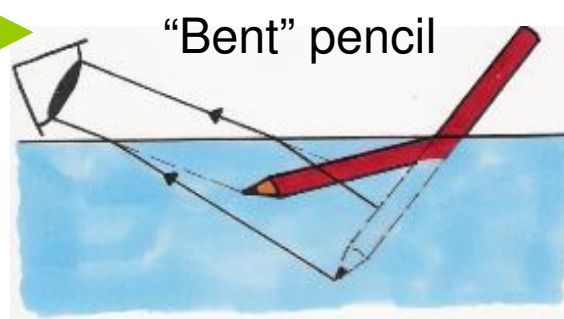
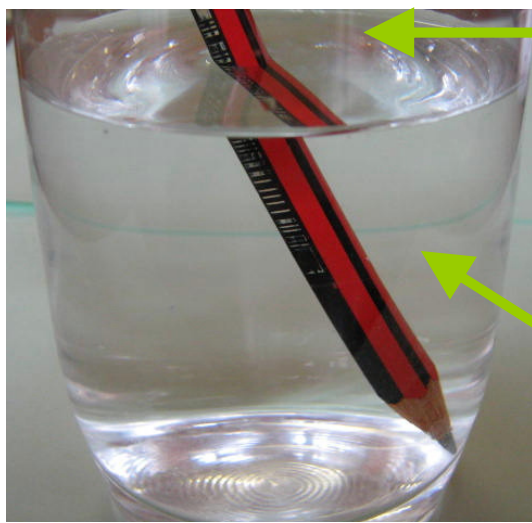
Then

$$\frac{\sin \theta_i}{\sin \theta_t} = \left( \frac{\epsilon_2}{\epsilon_1} \right)^{1/2} = \frac{\eta_1}{\eta_2}$$

Do not confuse n with  $\eta$

Since 
$$\eta = \sqrt{\frac{\mu}{\epsilon}}$$

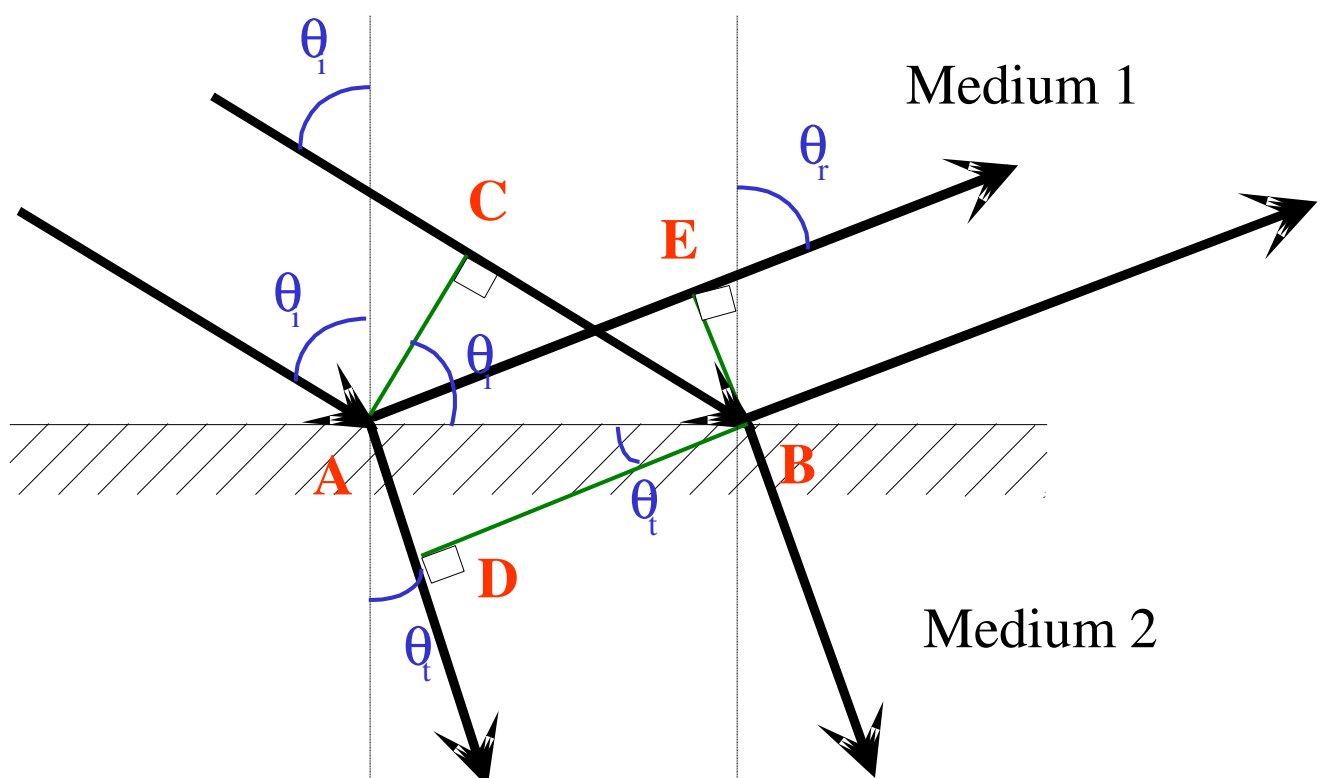
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Examples of refraction

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### II.3.2.2 Snell's law of reflection



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By a similar argument it can be shown that the angle of reflection is equal to the angle of incidence

$$AE = CB$$

but

$$CB = AB \sin \theta_i$$

$$AE = AB \sin \theta_r$$

➡  $\sin \theta_i = \sin \theta_r$

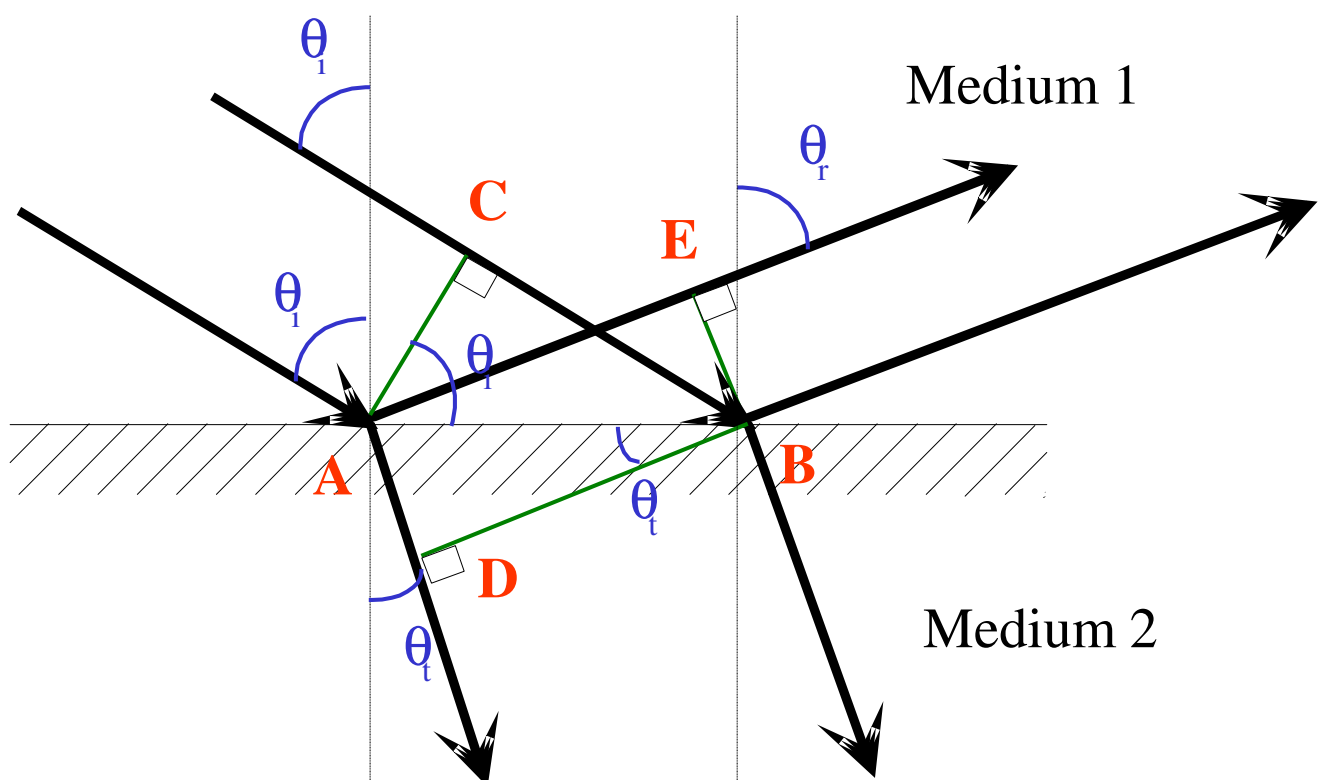
$$\theta_r = \theta_i$$

Angle of reflection is equal to angle of incidence

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### II.3.2.3 Incident and reflected power

The next step is to consider the power striking the surface AB and equate it to the power leaving that surface



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$$P_i = \frac{E_i^2}{2\eta_1} \cos \theta_i$$

$$P_r = \frac{E_r^2}{2\eta_1} \cos \theta_r$$

$$P_t = \frac{E_t^2}{2\eta_2} \cos \theta_t$$

We get: 
$$\frac{E_i^2}{\eta_1} \cos \theta_i = \frac{E_r^2}{\eta_1} \cos \theta_r + \frac{E_t^2}{\eta_2} \cos \theta_t$$

For dielectrics we assume  $\mu_1 = \mu_2 = \mu_0$

Remembering that 
$$\left( \frac{\epsilon_2}{\epsilon_1} \right)^{\frac{1}{2}} = \frac{\eta_1}{\eta_2} \quad \cos \theta_r = \cos \theta_i$$

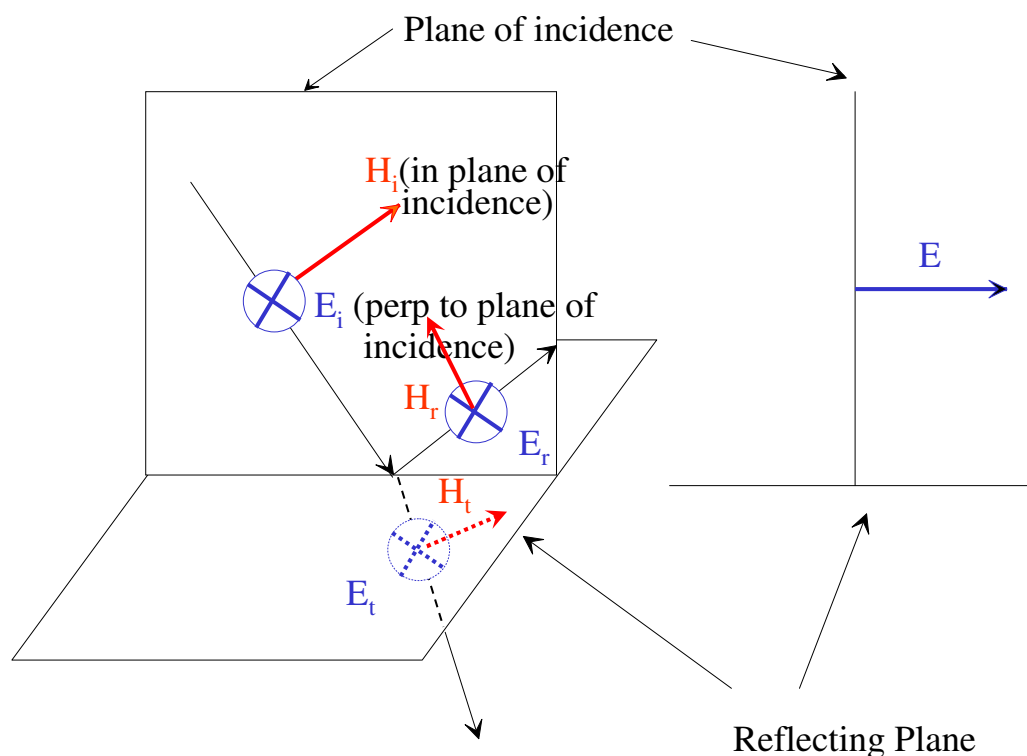
→ 
$$\frac{E_r^2}{E_i^2} = 1 - \left( \frac{\epsilon_2}{\epsilon_1} \right)^{\frac{1}{2}} \frac{E_t^2 \cos \theta_t}{E_i^2 \cos \theta_i} \quad (6.3)$$

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### II.3.3 Perpendicularly polarised waves

In these waves the electric field is perpendicular to the plane of incidence i.e. parallel to the boundary between the two media.

Light with perpendicular polarization is called **s-polarized** (s=*senkrecht*, perpendicular in German).



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From equation 6.3:

$$\frac{E_r^2}{E_i^2} = 1 - \left( \frac{\epsilon_2}{\epsilon_1} \right)^{\frac{1}{2}} \frac{E_t^2 \cos \theta_t}{E_i^2 \cos \theta_i} = 1 - k \frac{E_t^2}{E_i^2}$$

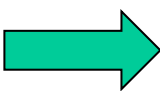
Where  $k = \left( \frac{\epsilon_2}{\epsilon_1} \right)^{\frac{1}{2}} \frac{\cos \theta_t}{\cos \theta_i}$

But, in this case  $E_i + E_r = E_t$

We then get the following:

$$\frac{E_r^2}{E_i^2} = 1 - k \frac{(E_r + E_i)^2}{E_i^2}$$

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  $\left( \frac{E_r}{E_i} \right)^2 (1 + k) + \left( \frac{E_r}{E_i} \right) (2k) + (k - 1) = 0$

This has the same form as

$$ax^2 + bx + c = 0$$

The positive solution is  $\frac{E_r}{E_i} = \frac{1 - k}{1 + k}$

Thus  $\frac{E_r}{E_i} = \frac{\epsilon_1^{1/2} \cos \theta_i - \epsilon_2^{1/2} \cos \theta_t}{\epsilon_1^{1/2} \cos \theta_i + \epsilon_2^{1/2} \cos \theta_t}$

or  $\frac{E_r}{E_i} = \frac{n_1 \cos \theta_i - n_2 \cos \theta_t}{n_1 \cos \theta_i + n_2 \cos \theta_t}$

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This expression contains the angle of the transmitted wave

We can use Snell's law to obtain a more useful expression which contains only the angle of incidence

From Snell's Law: 
$$\frac{\sin \theta_i}{\sin \theta_t} = \left( \frac{\epsilon_2}{\epsilon_1} \right)^{1/2}$$

But 
$$\cos \theta_t = \sqrt{1 - (\sin \theta_t)^2} = \sqrt{1 - \frac{\epsilon_1}{\epsilon_2} (\sin \theta_i)^2}$$

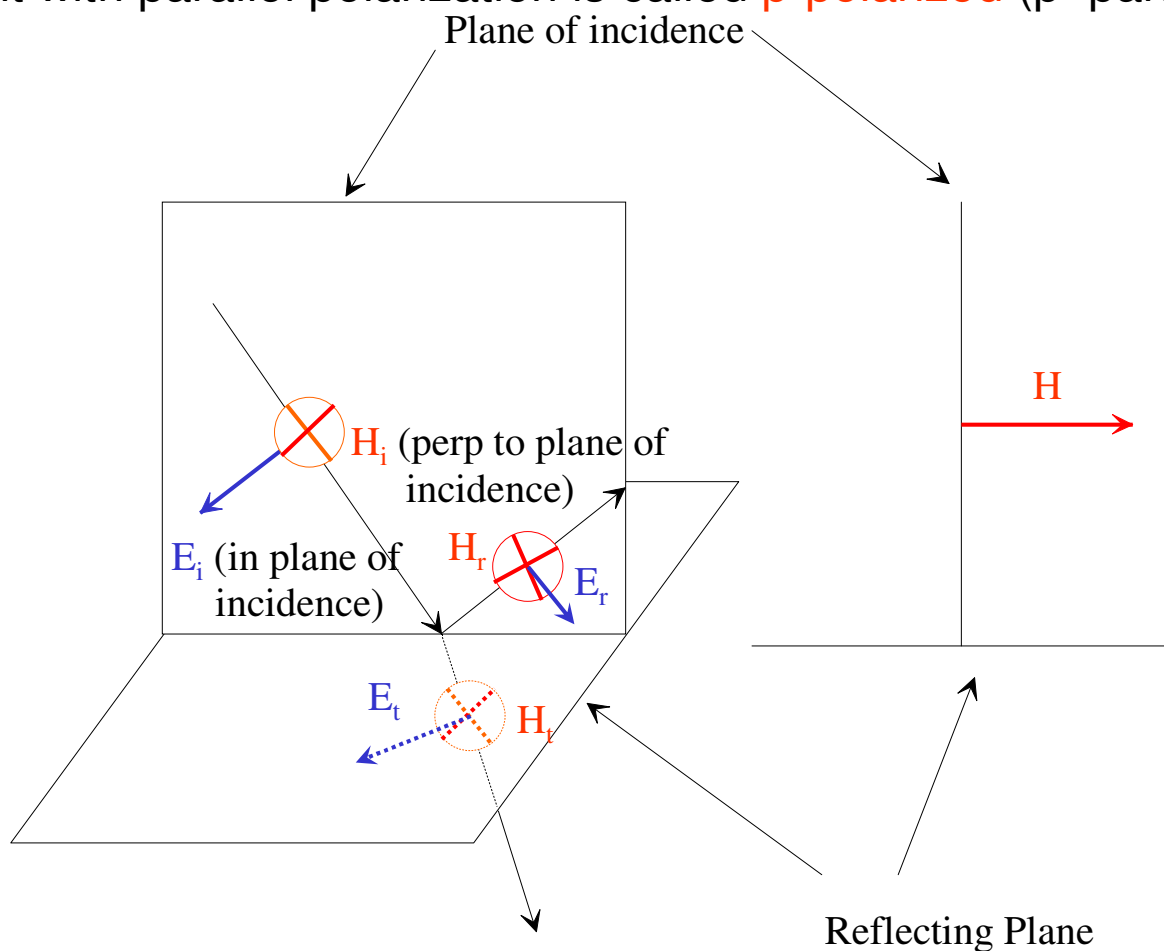
Then

$$\frac{E_r}{E_i} = \frac{\cos \theta_i - \left( \frac{\epsilon_2}{\epsilon_1} - \sin^2 \theta_i \right)^{1/2}}{\cos \theta_i + \left( \frac{\epsilon_2}{\epsilon_1} - \sin^2 \theta_i \right)^{1/2}} = \frac{n_1 \cos \theta_i - n_2 \sqrt{1 - \left( \frac{n_1}{n_2} \sin \theta_i \right)^2}}{n_1 \cos \theta_i + n_2 \sqrt{1 - \left( \frac{n_1}{n_2} \sin \theta_i \right)^2}} \quad (6.4)$$

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### II.3.4 Parallel polarised waves

Light with parallel polarization is called **p-polarized** (p=parallel).



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In this case E is no longer parallel to the reflecting plane. The boundary condition applies to the component of E parallel to the reflecting plane:

$$E_i \cos \theta_i - E_r \cos \theta_i = E_t \cos \theta_t$$

Following through the algebra our expression for the ratio of reflected to incident waves becomes:

$$\frac{E_r}{E_i} = \frac{\epsilon_2^{1/2} \cos \theta_i - \epsilon_1^{1/2} \cos \theta_t}{\epsilon_2^{1/2} \cos \theta_i + \epsilon_1^{1/2} \cos \theta_t} = \frac{n_2 \cos \theta_i - n_1 \cos \theta_t}{n_2 \cos \theta_i + n_1 \cos \theta_t} \quad (6.5)$$

$$\frac{E_r}{E_i} = \frac{\left(\frac{\epsilon_2}{\epsilon_1}\right) \cos \theta_i - \left(\frac{\epsilon_2}{\epsilon_1} - \sin^2 \theta_i\right)^{1/2}}{\left(\frac{\epsilon_2}{\epsilon_1}\right) \cos \theta_i + \left(\frac{\epsilon_2}{\epsilon_1} - \sin^2 \theta_i\right)^{1/2}} = \frac{n_2 \cos \theta_i - n_1 \sqrt{1 - \left(\frac{n_1}{n_2} \sin \theta_i\right)^2}}{n_2 \cos \theta_i + n_1 \sqrt{1 - \left(\frac{n_1}{n_2} \sin \theta_i\right)^2}}$$

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Note: normal incidence corresponds to  $\theta_i=0$

$$\theta_i = 0 \Rightarrow \frac{E_r}{E_i} = \frac{n_2 - n_1}{n_2 + n_1}$$

(6.5) is similar to equation (6.4), but in (6.5) the numerator can become zero, i.e. no reflected wave

The angle at which this occurs is known as the **Brewster Angle**

Setting the numerator of (6.5) equal to zero we get

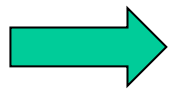
$$\text{Remembering that } \cos \theta_i = (1 - \sin^2 \theta_i)^{1/2} \quad \sin^2 \theta_i = \frac{\tan^2 \theta_i}{1 + \tan^2 \theta_i}$$

$$\tan(\theta_B) = \left(\frac{\epsilon_2}{\epsilon_1}\right)^{1/2} = \frac{n_2}{n_1} \quad \theta_B \text{ is called Brewster Angle}$$

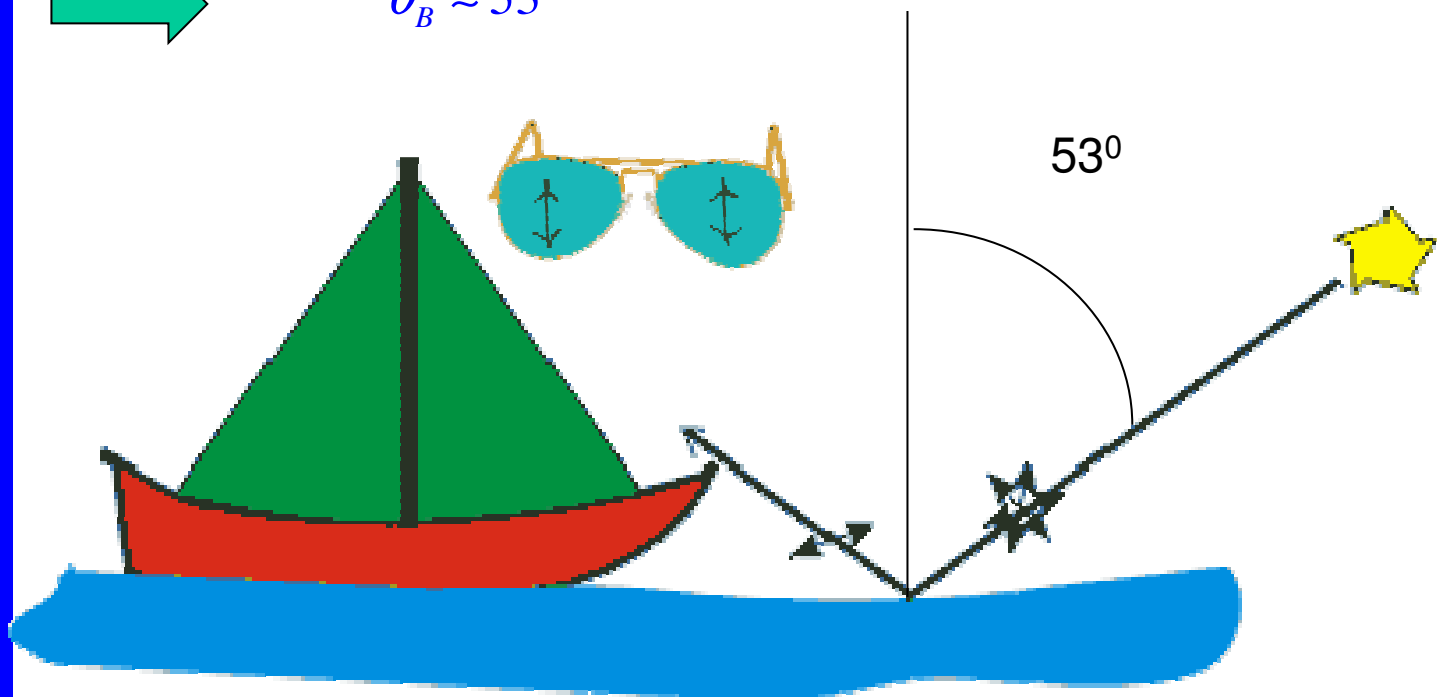
Note (6.4) can go to zero only for  $n_1=n_2$ : i.e. travel in the same medium 167

Zero reflection at the Brewster angle explains why polarised sunglasses cut down reflections

Let us consider the air ( $n_1=1$ ) water ( $n_2=1.33$ ) interface



$$\theta_B \approx 53^\circ$$



A rule of thumb: polarized filters limit the glare from calm waters for a sun altitude between 30 and 60 degrees

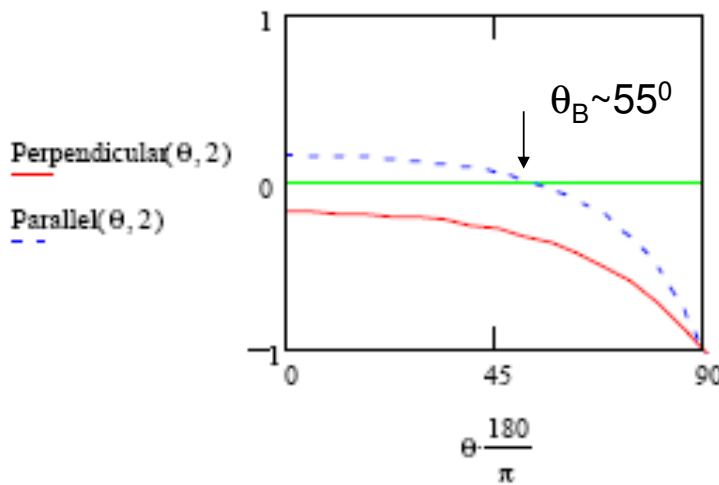
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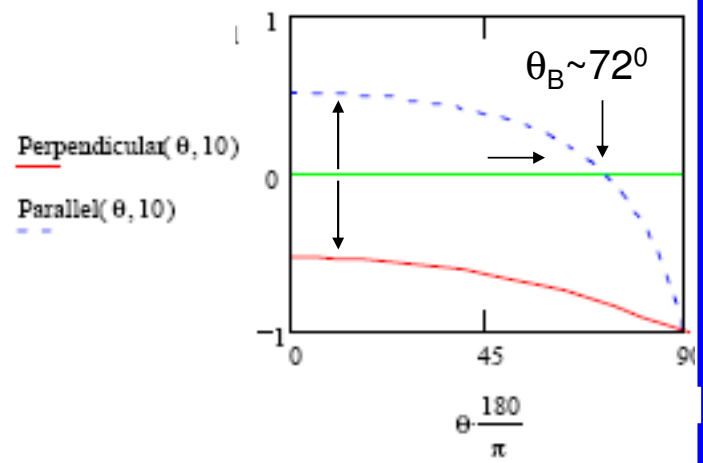
## II.3.5 Comparison between reflection of parallel and perpendicularly polarised waves

Graphs of  $\frac{E_r}{E_i}$  as a function of angle of incidence



a) Permittivity Ratio of 2

$$\frac{\epsilon_2}{\epsilon_1} = 2$$



b) Permittivity Ratio of 10

$$\frac{\epsilon_2}{\epsilon_1} = 10$$

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The graphs show the values of  $\frac{E_r}{E_i}$  for two permittivity ratios

As the ratio  $\frac{\epsilon_2}{\epsilon_1}$  increases three effects can be seen

- 1) The Brewster angle increases
- 2) The value of  $\frac{E_r}{E_i}$  for the perpendicularly polarized wave tends to -1, i.e. perfect antiphase at all angles
- 3) The value of  $\frac{E_r}{E_i}$  for the parallel polarized wave tends to 1, i.e. perfect phase at all angles
- 4) When  $\frac{\epsilon_2}{\epsilon_1} \rightarrow \infty$  total reflection occurs at all angles of incidence.

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## II.3.6 Total internal reflection

The previous section showed graphs for  $\frac{\epsilon_2}{\epsilon_1} > 1$ . I.e. our wave is moving from a lower refractive index medium to a higher one

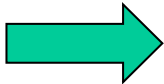
If instead  $\frac{\epsilon_2}{\epsilon_1} < 1$  then the phenomenon known as **total internal reflection** can occur

The term  $\left(\frac{\epsilon_2}{\epsilon_1} - \sin^2 \theta_i\right)$  in equations (6.4) or (6.5) can be negative

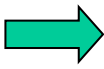
when  $\sin \theta_i \geq \left(\frac{\epsilon_2}{\epsilon_1}\right)^{1/2} = \frac{n_2}{n_1}$

The **critical angle** is defined as  $\sin \theta_{cr} = \left(\frac{\epsilon_2}{\epsilon_1}\right)^{1/2} = \frac{n_2}{n_1}$

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For  $\theta_i = \theta_{cr}$    $\frac{E_r}{E_i} = 1$

For  $\theta_i > \theta_{cr}$  the term  $\left(\frac{\epsilon_2}{\epsilon_1} - \sin^2 \theta_i\right)$  in (6.4) or (6.5) is negative

 its square root is imaginary and  $\frac{E_r}{E_i}$  is complex

$$\frac{E_r}{E_i} = \frac{A - iB}{A + iB} \quad \left(\frac{E_r}{E_i}\right)^* = \frac{A + iB}{A - iB}$$

  $\left|\frac{E_r}{E_i}\right|^2 = \left(\frac{E_r}{E_i}\right)\left(\frac{E_r}{E_i}\right)^* = 1$

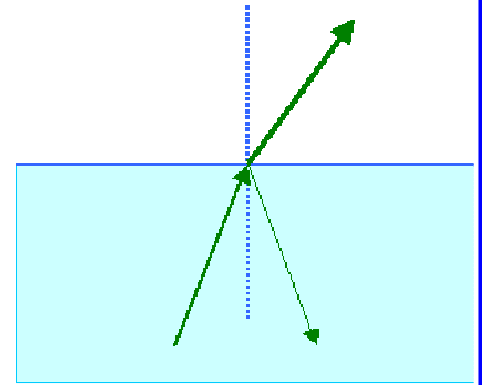
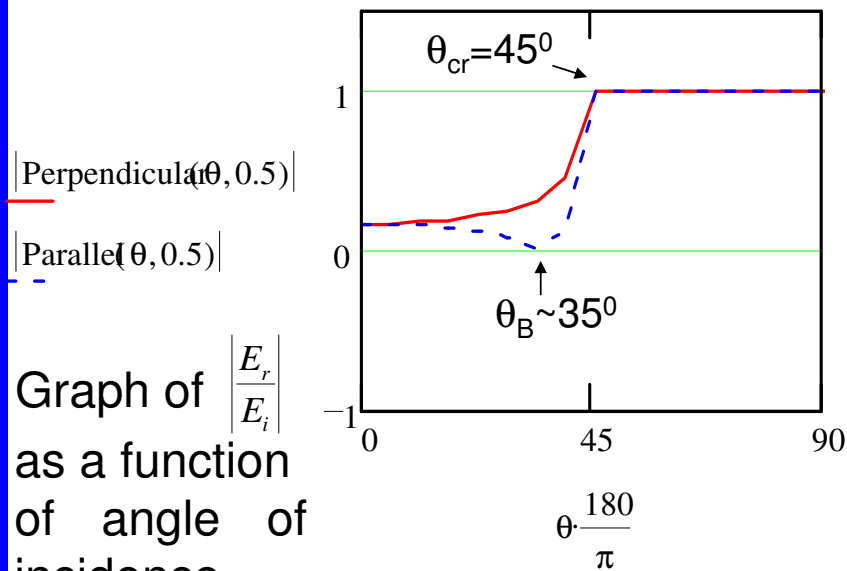
i.e. the magnitudes of the incident and reflected power are equal.

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This gives rise to *total internal reflection*: the incident wave is reflected at the boundary and no wave emerges from the higher refractive index medium

This is shown in the graph below at all angles of incidence where:

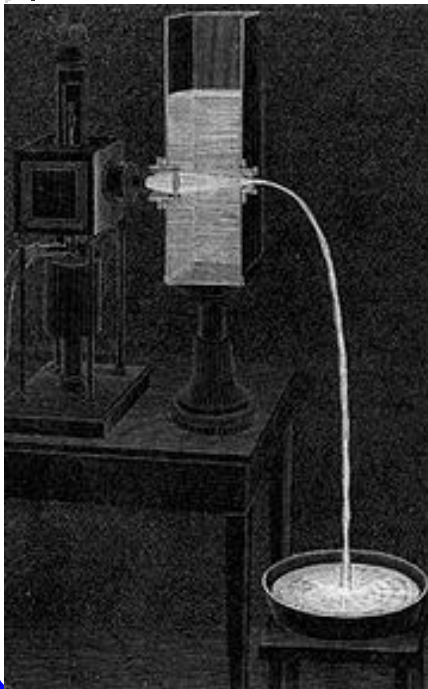
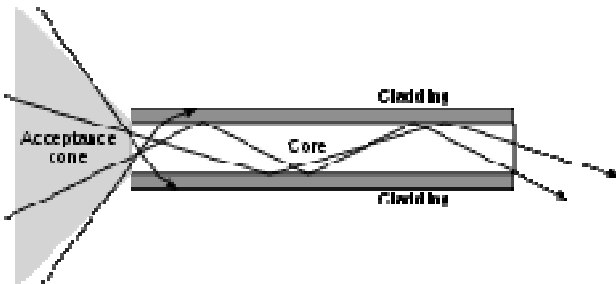
$$\sin^2 \theta \geq \frac{\epsilon_2}{\epsilon_1} \quad (\text{with } \frac{\epsilon_2}{\epsilon_1} = 0.5 \Rightarrow \theta \geq 45^\circ)$$



Permittivity Ratio of 1/2

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## Total internal reflection underpins optical fibre technology



1841  
Colladon

1854  
Tyndall

1965  
Charles K. Kao  
Nobel Prize 2009.



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### II.3.7 Example: reflected power

Diamond has  $\epsilon_r = 5.84$  &  $\mu_r = 1$ . What power fraction of light is reflected off an air/diamond surface for normal incidence?

Recalling that for transmission lines:

$$\bar{\rho}_L = \frac{\bar{V}_B}{\bar{V}_F} = \frac{\bar{Z}_L - Z_0}{\bar{Z}_L + Z_0}$$

Similarly for E-M waves, for the case where E and H are parallel to the reflecting plane:

$$\rho_r = \frac{E_{xr}}{E_{xi}} = \frac{\eta_{Diamond} - \eta_{Air}}{\eta_{Diamond} + \eta_{Air}} = \frac{\eta_{Diamond} - 1}{\eta_{Diamond} + 1} \quad \text{with} \quad \eta = \sqrt{\frac{\mu}{\epsilon}}$$

$$\text{Then } \frac{\eta_{Diamond}}{\eta_{Air}} = \frac{\sqrt{\frac{\mu_r \mu_0}{\epsilon_r \epsilon_0}}}{\sqrt{\frac{\mu_0}{\epsilon_0}}} = \sqrt{\frac{\mu_r}{\epsilon_r}} = \sqrt{\frac{1}{5.84}} \approx 0.41 \Rightarrow |\bar{\rho}_R|^2 = \left| \frac{0.41 - 1}{0.41 + 1} \right|^2 \approx 17.5\%$$

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## III Antennae and Radio Transmission

### Aims

To give a qualitative description of antennae and radio transmission

### Objectives

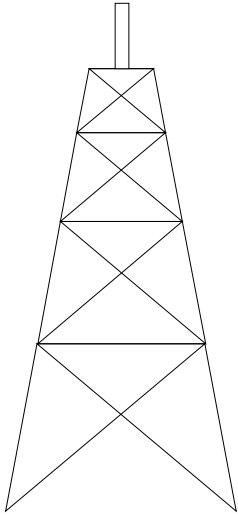
At the end of this section you should be able to recognise the main classes of antennae, and do simple calculations of radiation resistance, effective area, gain.

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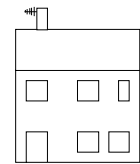
## III.1 Antennae

The aim of an antenna is to get signal power from the transmitter to the receiver circuit as efficiently as possible

Transmitter



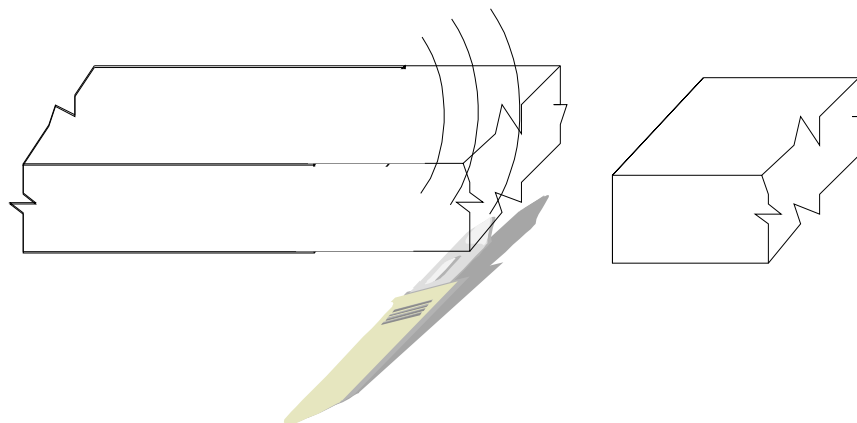
Receiver



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### III.1.1 Slot and aperture antennae

Almost any guide carrying an electromagnetic wave will radiate part of the wave if its end is open



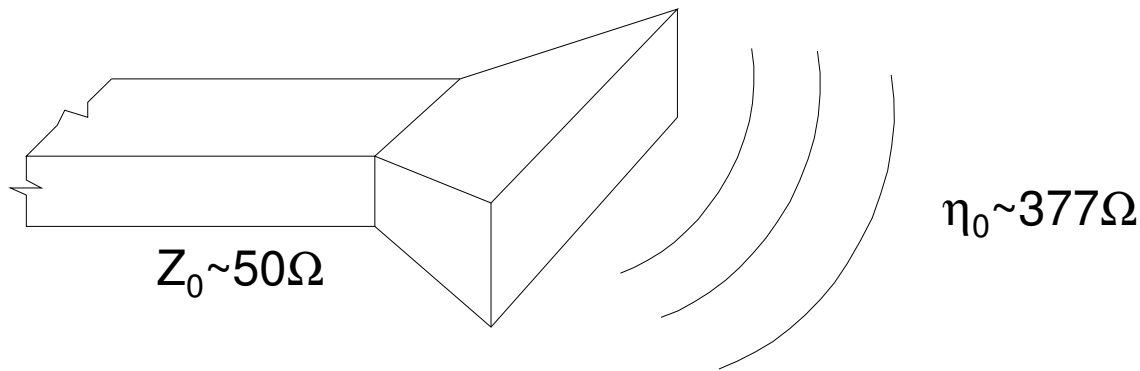
The electromagnetic waves in a guide will radiate if you chop its end off (very inefficient)

The ideal antenna sends as much radiation as possible in the desired direction and with the minimum of internal reflection.

179

### III.1.1.1 Horn antennae

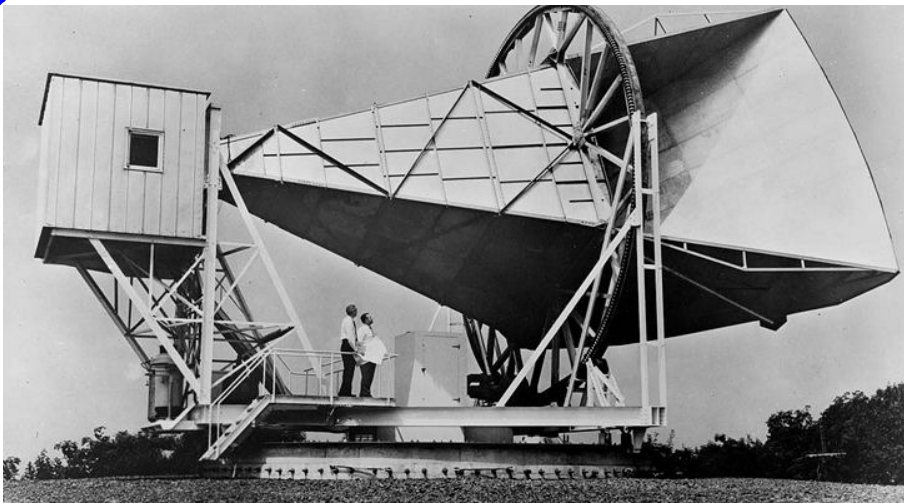
More wave is radiated if the end of the guide is flared



Bigger aperture so

- Less diffraction and more gain (directed power, see III.2)
- The launched wave is more similar to a plane wave
- There is a gradual change between the electrical wave, with characteristic impedance  $Z_0$ , and the radiated wave, with intrinsic impedance  $\eta_0$ , hence less reflection.

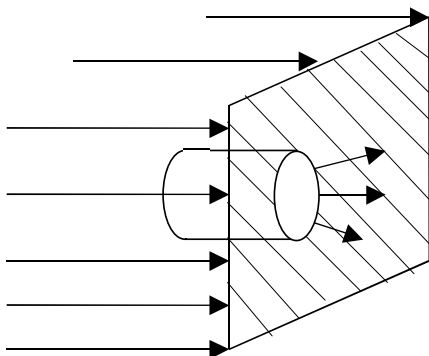
180



Horn antennae such as these work very well but they are bulky, therefore unwieldy

1965 Penzias & Wilson used this horn antenna to detect the background microwave radiation of the universe. 1978 Nobel Prize Physics

### III.1.1.2 Laser



A laser can be thought of as an aperture antenna and the output roughly approximates to a plane wave.

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## III.1.2 Dipole antennae

### III.1.2.1 Half-wave dipole

If the end of a transmission line is left open circuit, the current at the end is 0 and the reflection coefficient is 1

$$\rho_L = \frac{\bar{Z}_L - Z_0}{\bar{Z}_L + Z_0}$$

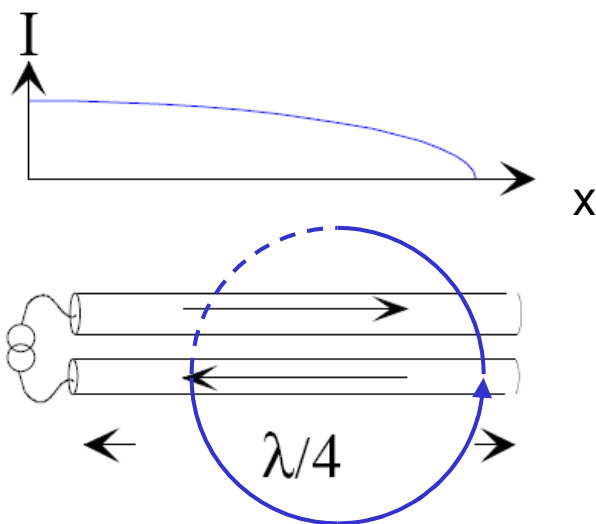
if  $Z_L \rightarrow \infty$  then  $\rho_L \rightarrow 1$

In this case, the telegrapher's equations show that  $\frac{1}{4}$  of a wavelength back from the load the voltage must be 0

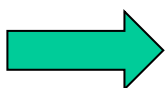
$$Z_{b=\frac{\lambda}{4}} = \frac{Z_0^2}{Z_L} = 0 \quad \rightarrow \quad V_{b=\frac{\lambda}{4}} = Z_{b=\frac{\lambda}{4}} I = 0$$

182

$\Rightarrow$  a current source is needed to drive the line



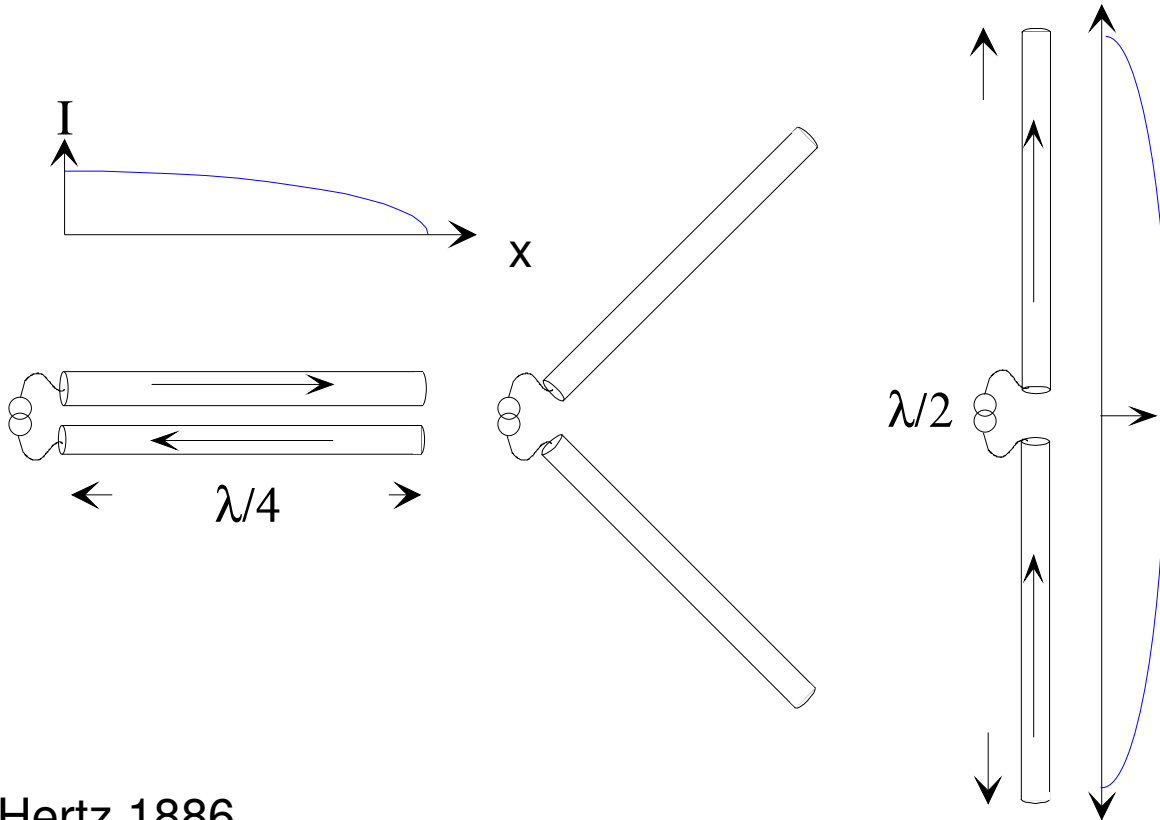
But 
$$\oint_c \underline{H} \cdot d\underline{l} = \int_s (\underline{J}) \cdot d\underline{S} = I + (-I) = 0$$



No radiated field

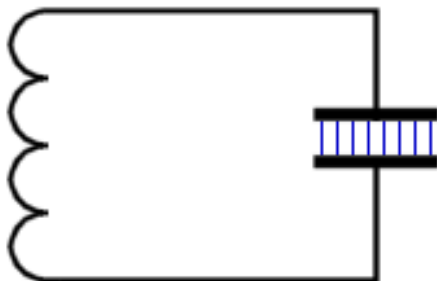
183

Opening out the two bars of the transmission line has little effect on the current distribution and the exposed oscillating current radiates an electromagnetic wave



Hertz 1886

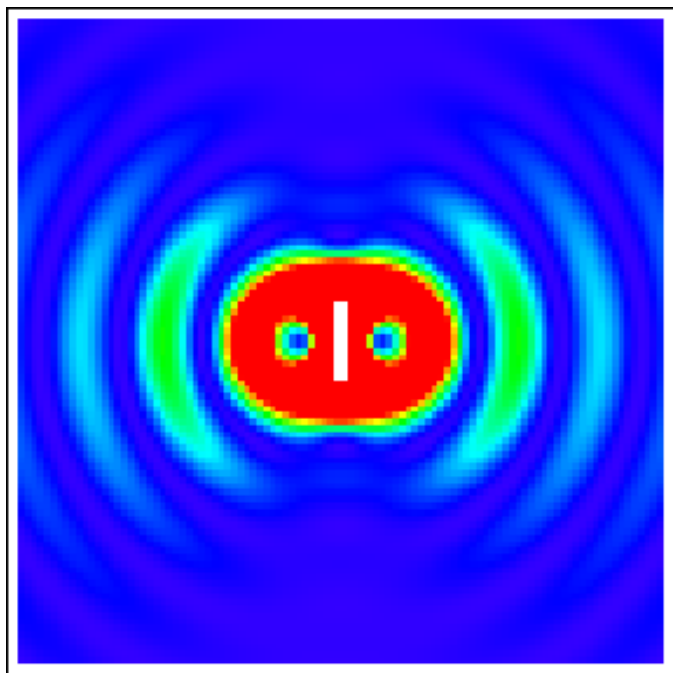
184



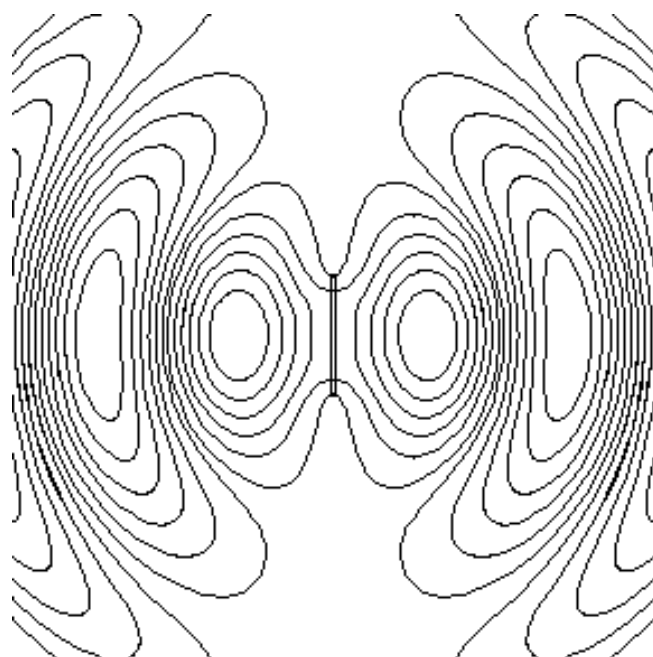
GHz half-wave dipole Antenna

185

Power distribution.



Electric field

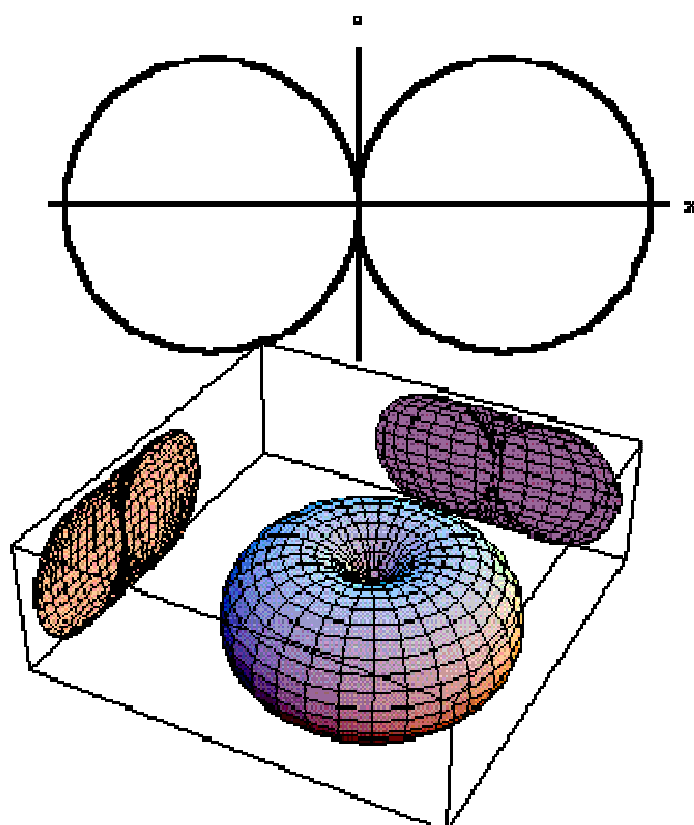
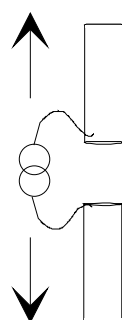


186

### III.1.2.2 Short dipole

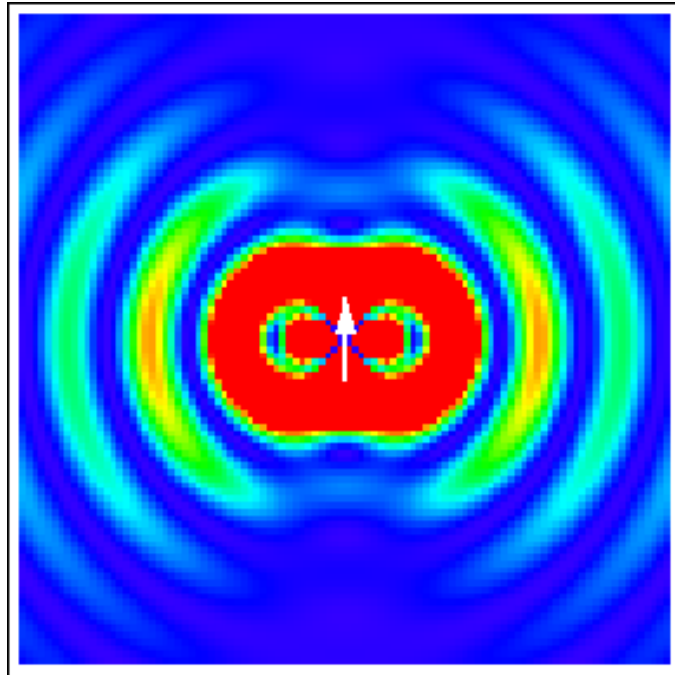
At long wavelengths a half-wave dipole becomes impractically big. A shorter dipole still radiates, but more of the wave is reflected back down the waveguide, since there is an input impedance

$$L \ll \lambda/2$$

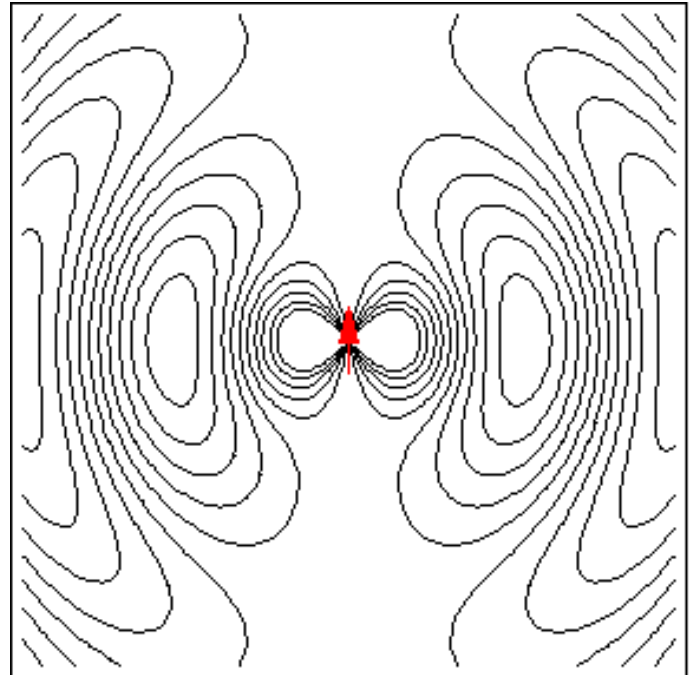


187

Power distribution

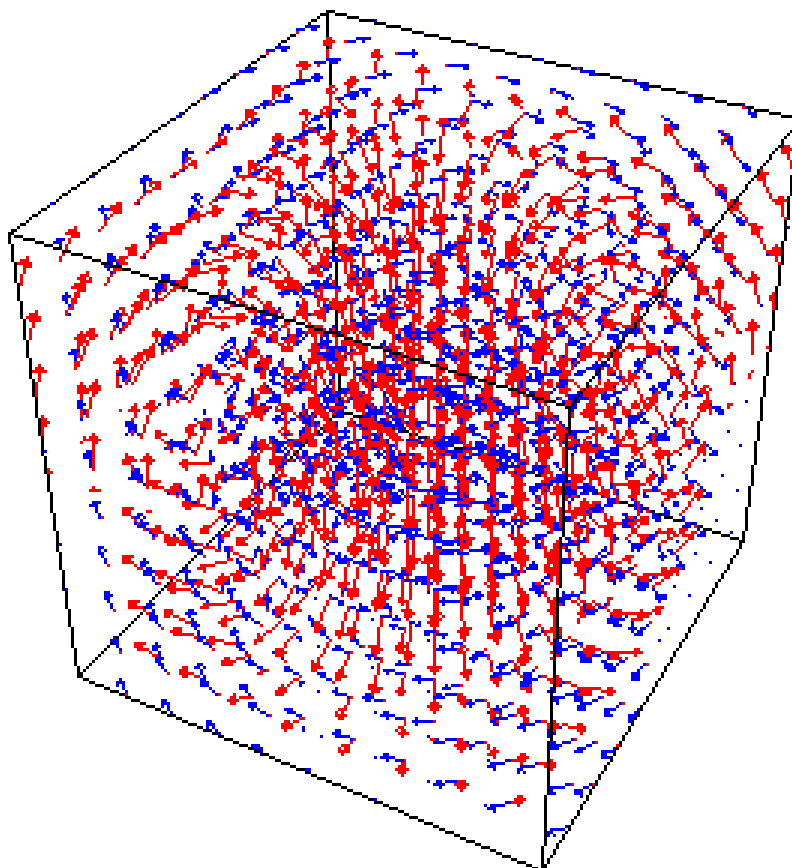


Electric field



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Electric and magnetic fields



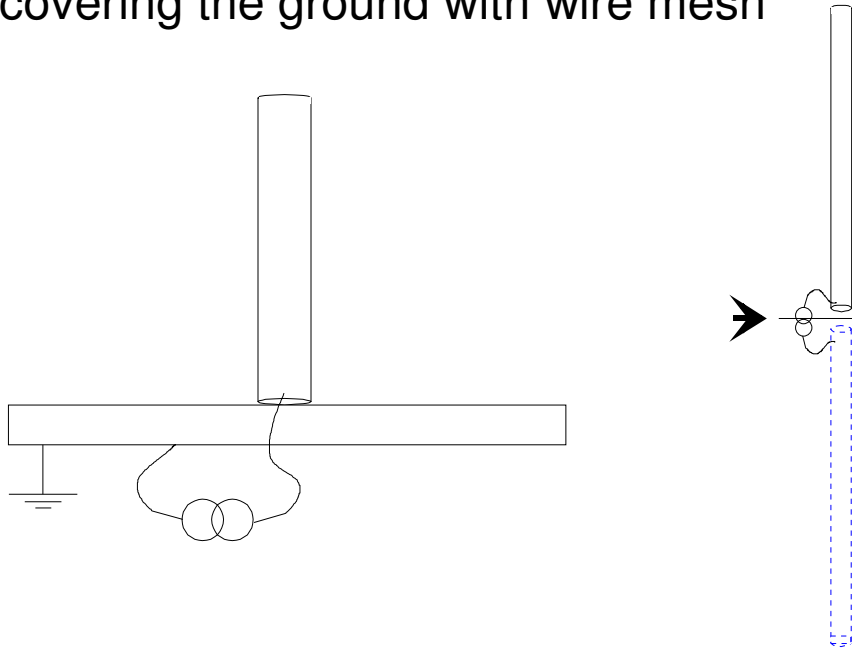
189

### III.1.2.3 Half dipole

Conductors reflect radio waves because  $E=0$  at the conductor (that is why mirrors are shiny)

A single dipole placed above a conductor radiates like one half of a dipole pair

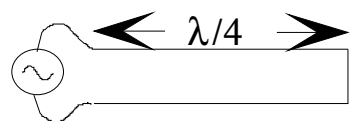
Long wave radio masts are like this, and can be formed by covering the ground with wire mesh



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### III.1.3 Loop antennae

The loop antenna is like the half-wave dipole except that it behaves like a transmission line whose end is a short circuit and is therefore driven by a voltage source



Short Circuit a  $\lambda/4$  transmission line..

If the end of a transmission line is a short circuit,  $Z_L=0$

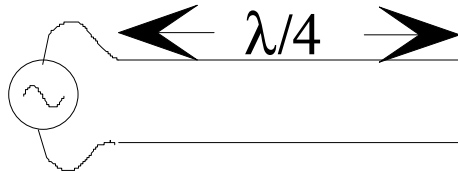
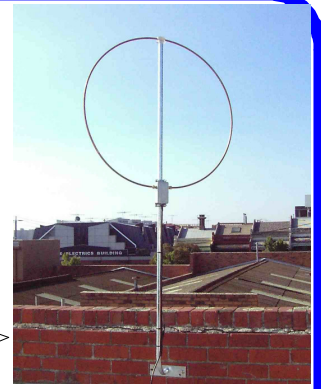
➡  $\rho_L = -1$

The telegrapher's equations show that  $\frac{1}{4}$  of a wavelength back from the load the current must be 0.

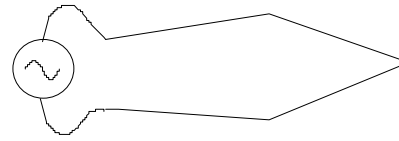
191

$$Z_{b=\frac{\lambda}{4}} = \frac{Z_0^2}{Z_L} \rightarrow \infty \quad \Rightarrow \quad I_{b=\frac{\lambda}{4}} = \frac{V_{b=\frac{\lambda}{4}}}{Z_{b=\frac{\lambda}{4}}} \rightarrow 0$$

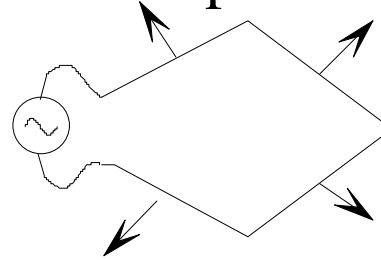
so a voltage source is needed to drive the wave



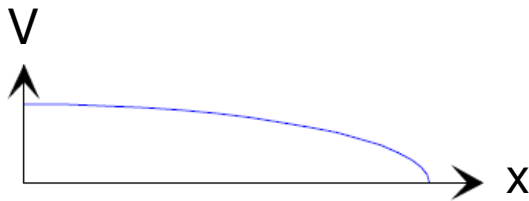
Short Circuit a  $\lambda/4$  transmission line..



then open it out..

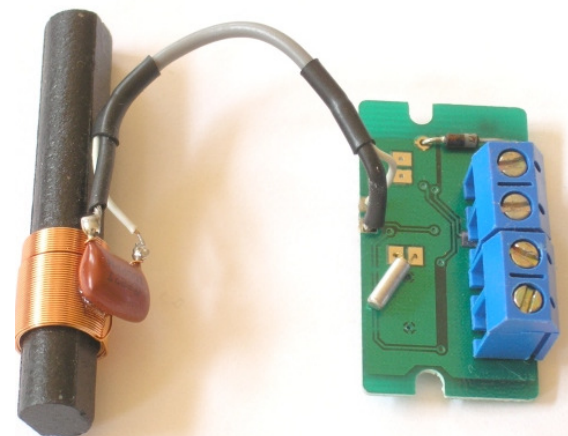
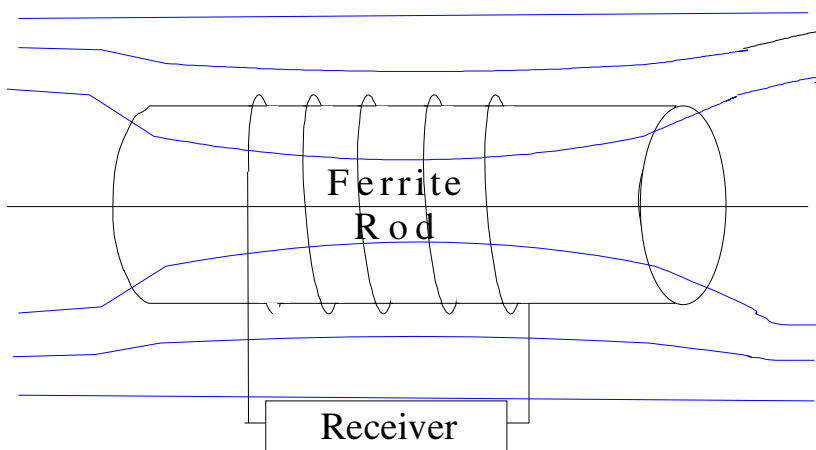


To get a loop antenna



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Inside a portable radio you will find a ferrite rod wound with copper wire. This is the long wave antenna. It has several loops and a ferrite core. The core concentrates the electromagnetic waves into the antenna and the whole arrangement is essentially half of a transformer

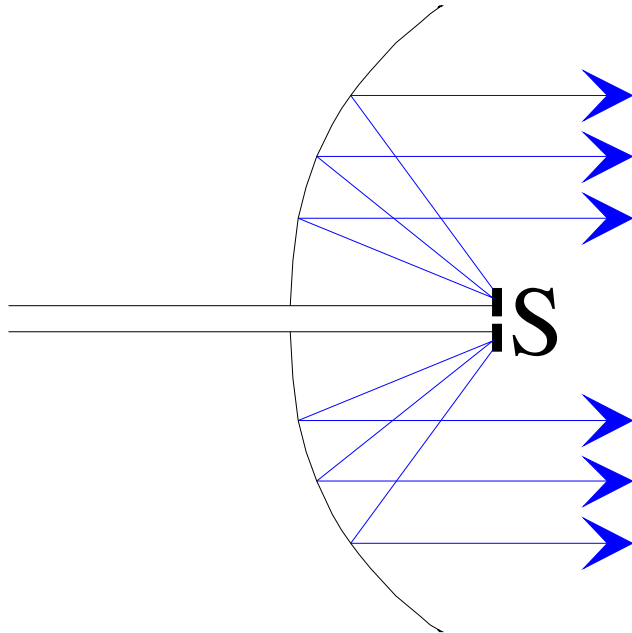


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### III.1.4 Reflector antennae

Another way of concentrating the electromagnetic waves is using a parabolic mirror

Placing a source, S, at the focus of a paraboloid can result in a very directive beam



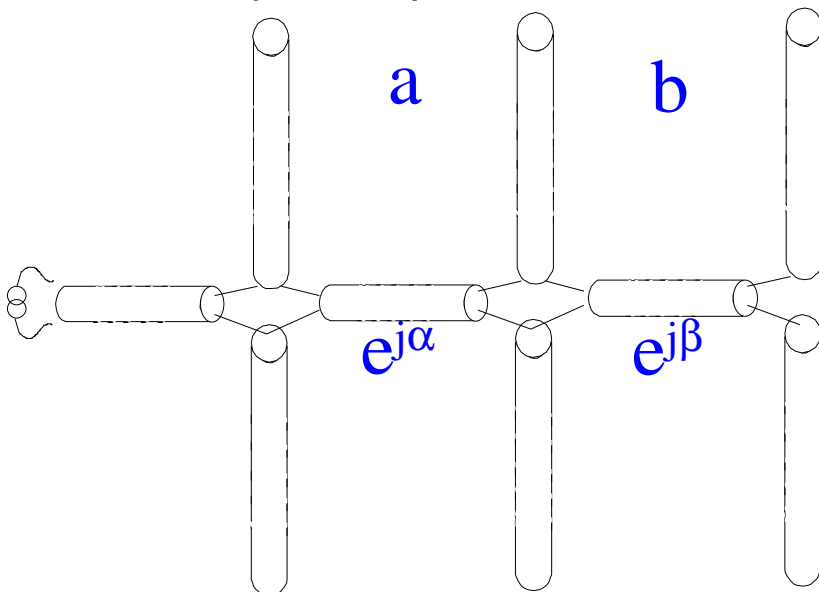
194

### III.1.5 Array antennae

Antennae can also be joined into an array

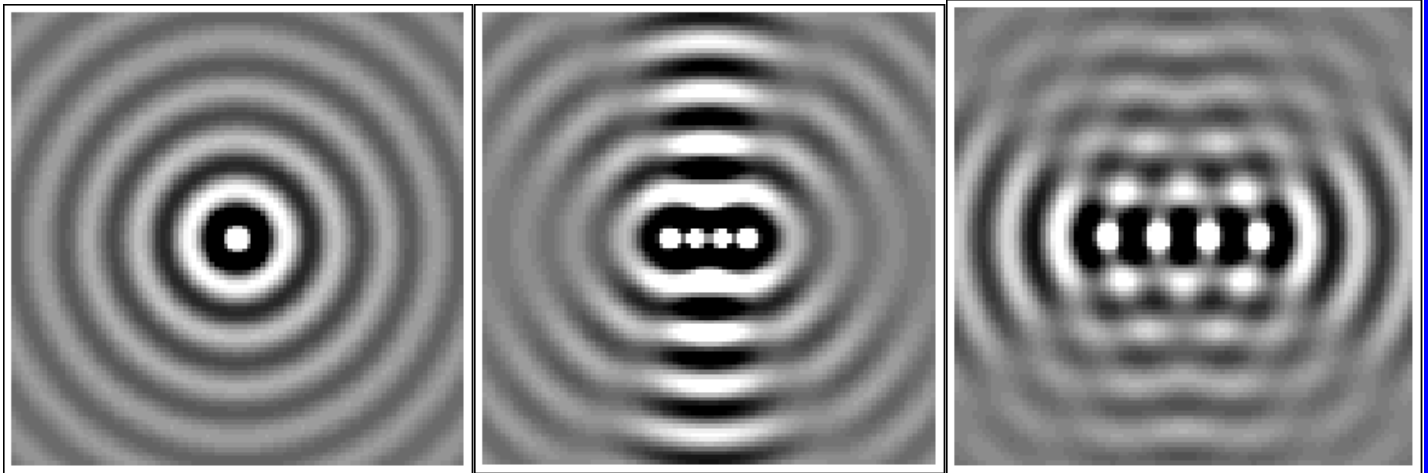
The array must be correctly designed so that the signals combine in phase

i.e. that they add up rather than cancel out



Superposition effects result in a highly directional wave as long as the spacings (a,b) and the phases ( $e^{j\alpha}$ ,  $e^{j\beta}$ ) are correct

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## III.2 Radio transmission

### III.2.1 Radiation resistance

Antennae emit power, so they can be modelled as resistors:

The radiation resistance,  $R_a$ , of an antenna is that resistance which in place of the antenna would dissipate as much power as the antenna radiates

$$R_a = \frac{P_{ant}}{(I_{rms})^2} \quad (8.1)$$

where 
$$P_{ant} = \int_s k I_{ant}^2 d\underline{S} = k \int_s I_{ant}^2 d\underline{S} \quad (8.2)$$

and 
$$k I_{ant}^2 = \frac{1}{2} \text{Re} \left\{ \overline{\underline{E}} \times \overline{\underline{H}}^* \right\}$$

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The power is calculated by integrating over a surface far from the radiating antenna which encloses the current carrying portion

For example, a sphere at a distance  $r$  away from the axis of a wire carrying a current  $I_{ant}$

As the distance  $r$  increases  $E$  varies as  $E_0/r$  and  $H$  as  $E_0/r\eta$

These are termed the **far field values** of the electromagnetic radiation from the antenna

The intensity of power from a real antenna is not uniform. It has a 3 dimensional pattern, which is termed **the radiation pattern**

**Example: consider a half dipole antenna with:**

$$P_{ant} = 36.5 I_{ant}^2 ; \quad I_{rms} = \frac{I_{ant}}{\sqrt{2}} \quad \longrightarrow \quad R_a = \frac{36.5 I_{ant}^2}{\left( \frac{I_{ant}}{\sqrt{2}} \right)^2} = 73 \Omega.$$

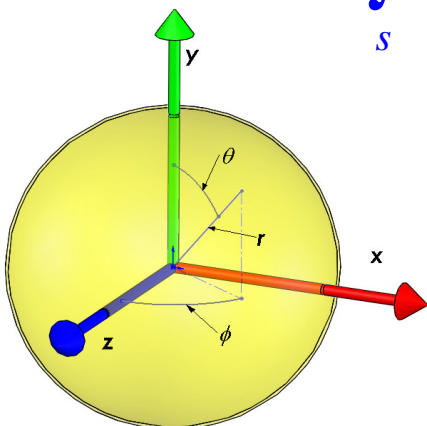
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## III.2.2 Gain

The gain,  $G$ , of an antenna is the factor by which its maximum radiated intensity exceeds that of an isotropic antenna if they emit equal power from an equal distance

$$G = \frac{\frac{1}{2} \text{Re} \left\{ \underline{\underline{E}}_{ant}(r, \theta, \phi) \times \overline{\underline{\underline{H}}_{ant}^*}(r, \theta, \phi) \right\}_{Max}}{\frac{1}{2} \text{Re} \left\{ \underline{\underline{E}}_{iso}(r, \theta, \phi) \times \overline{\underline{\underline{H}}_{iso}^*}(r, \theta, \phi) \right\}} = \frac{k I_{ant}^2}{R_a I_{iso}^2 / 4 \pi r^2} \quad (8.3)$$

Provided that  $\int_S \underline{\underline{Int}}_{antenna} \cdot \underline{\underline{dS}} = \int_S \underline{\underline{Int}}_{isotropic} \cdot \underline{\underline{dS}}$



An isotropic antenna is a hypothetical device which radiates equally in all directions.

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### III.2.3 Effective area

The effective area,  $A_{\text{eff}}$ , of an antenna is that area of wavefront whose power equals that received from the wavefront by the antenna

$$A_{\text{eff}} = \frac{\text{Power collected by antenna}}{\text{Wave intensity (i.e. power / area) into antenna}}$$

Note: even though the area of radio wave intercepted by a parabolic dish antenna is obvious, it is not in other cases

For example: in principle a half wave dipole could have no area at all and yet still receive power from a radio wave. Hence the need to define an effective area.

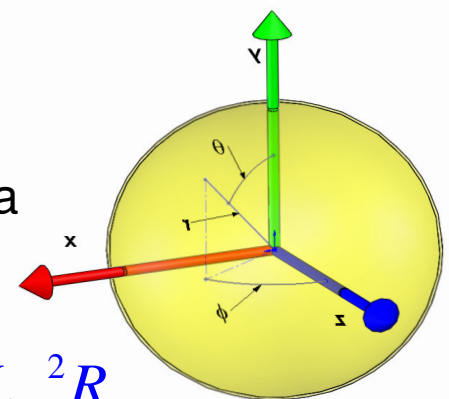
200

### III.2.4 Example: Power transmission

If two half-wave dipoles are 1 km apart and one is driven with 0.5 amps (RMS) at 300 MHz, what power is received by the other ?

$$G = 1.64, R_a = 73 \, \Omega, A_{\text{eff}} = 0.13 \, \text{m}^2$$

Intensity  $r$  metres from an isotropic antenna  
= Transmitted power / ( $4\pi r^2$ )



$$\text{Intensity } r \text{ metres from this antenna} = G \frac{I_{\text{iso}}^2 R_a}{4\pi r^2}$$

Power received by receiving antenna = Intensity  $\cdot A_{\text{eff}}$

$$= G \frac{I_{\text{iso}}^2 R_a}{4\pi r^2} A_{\text{eff}} = 1.64 \cdot \frac{0.5^2 \cdot 73}{4\pi (1000)^2} \cdot 0.13 = 0.3 \, \mu\text{W}.$$

201

## Optional Examples

**O1-Negative refractive index**

**O2-Invisibility cloak**

**O3-Hard disk drives**

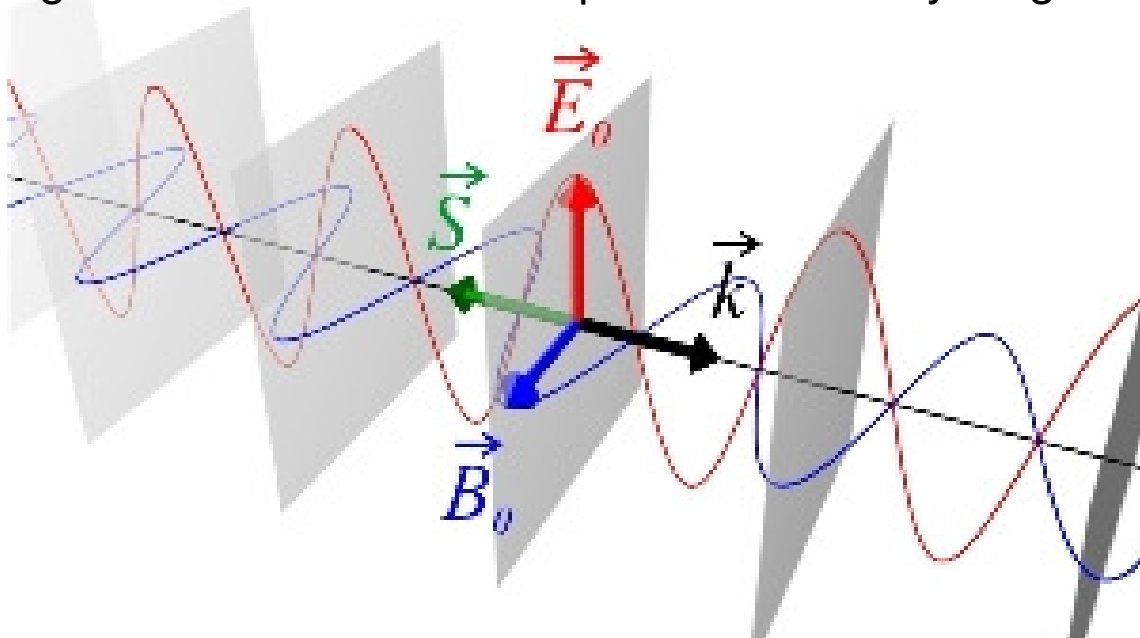
**O4-Transparent conductors**

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## O1-Negative refractive index

In 1967 Viktor Veselago theoretically investigated the properties of media with a negative permittivity  $\epsilon$  together with negative permeability  $\mu$  in the same frequency range

He predicted that the wave vector of a wave propagating through such a medium is antiparallel to its Poynting vector



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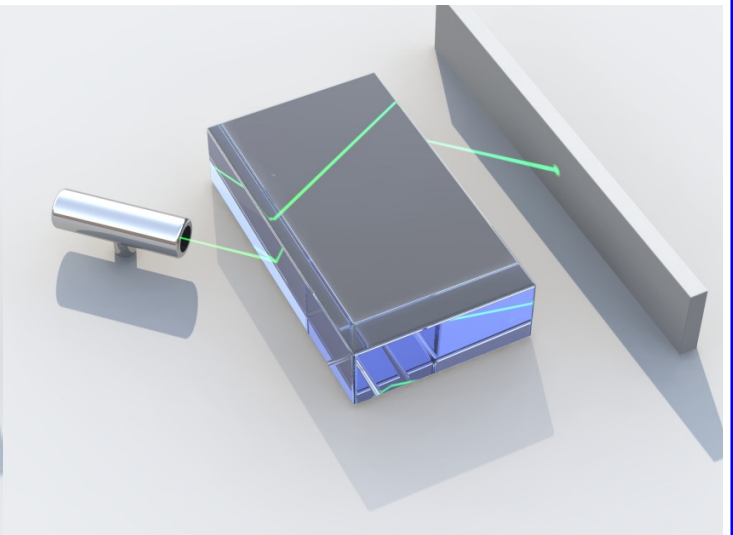
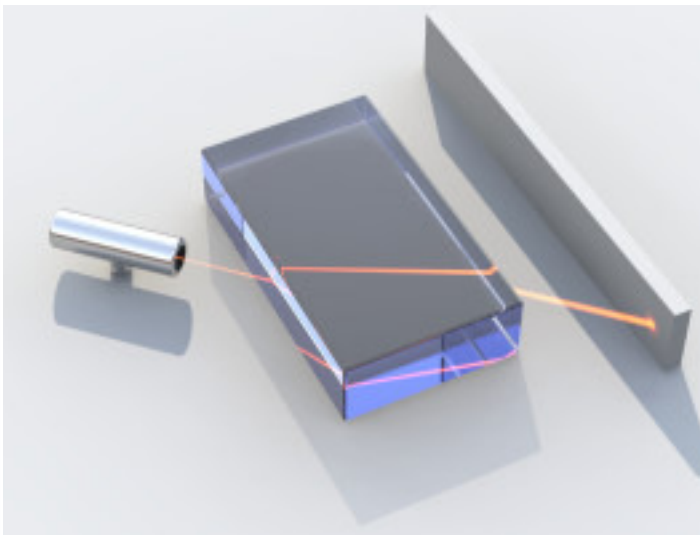
This has far-reaching consequences:

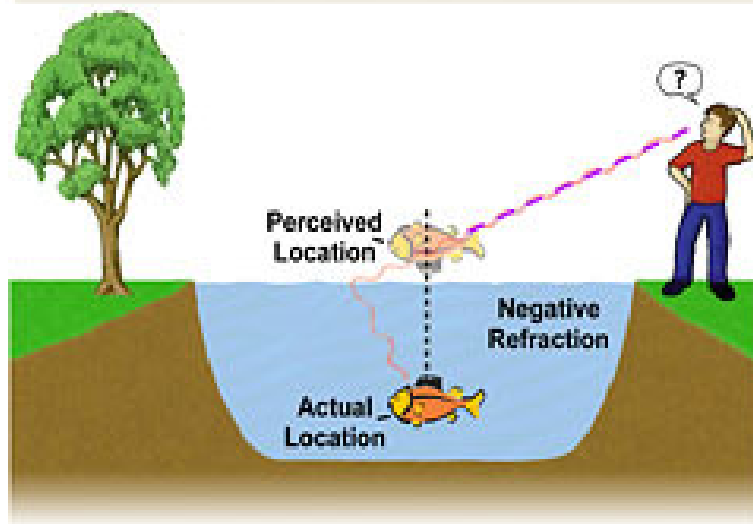
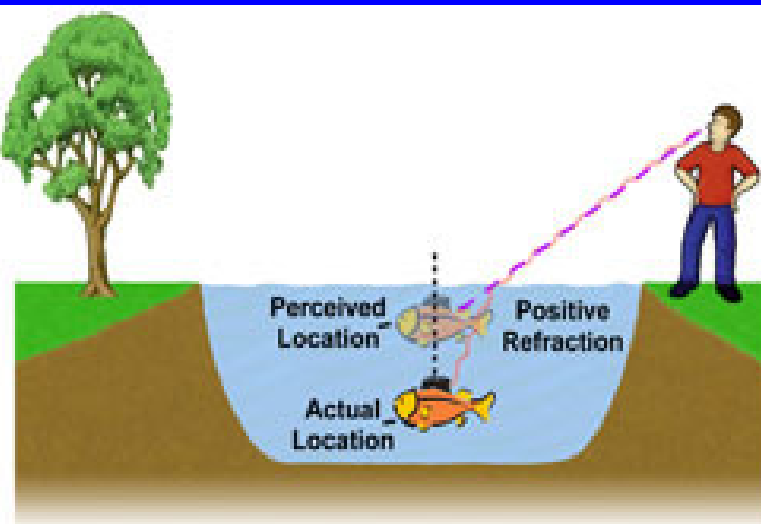
A wave impinging from vacuum onto the surface of such a medium under an angle with respect to the surface normal will be refracted towards the "wrong" side of the normal, i.e., **we obtain negative refraction**

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### Positive Refractive Index

### Negative Refractive Index



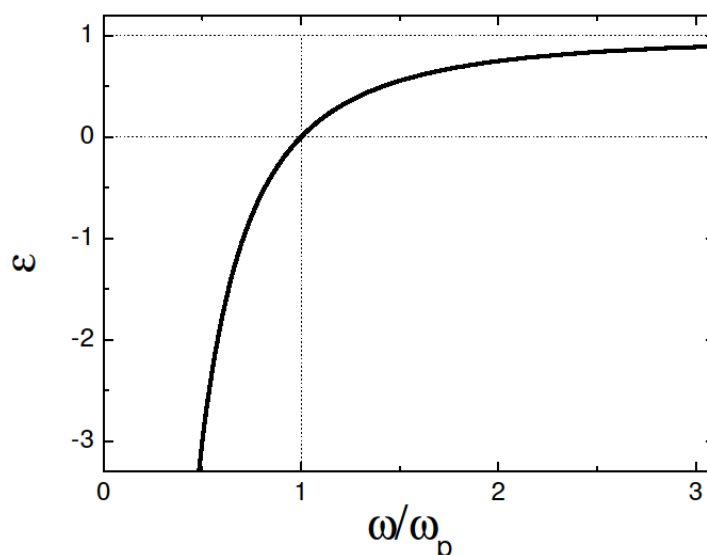


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Negative permittivity  $\epsilon$  is not unusual, and occurs in metals from zero frequency to the plasma frequency

$$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2} \quad \omega_p = \sqrt{\frac{4\pi n e^2}{m}} \quad \text{Plasma Frequency}$$

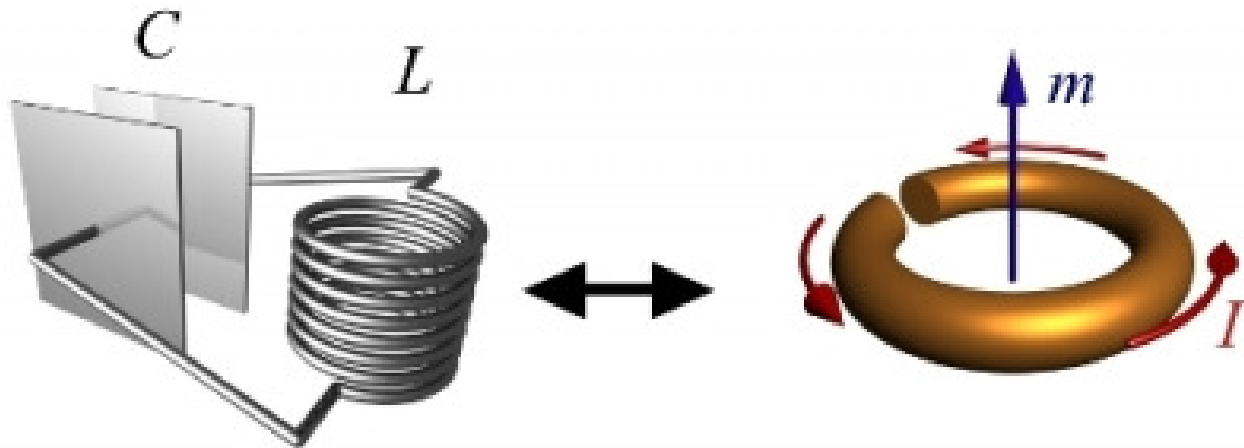
With  $n$  electron charge density and  $m$  electron mass



However a large magnetic response, in general, and a negative permeability  $\mu$  at optical frequencies, in particular, do not occur in natural materials

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A negative index of refraction can be implemented by an array of metallic split-ring resonators (SRR)

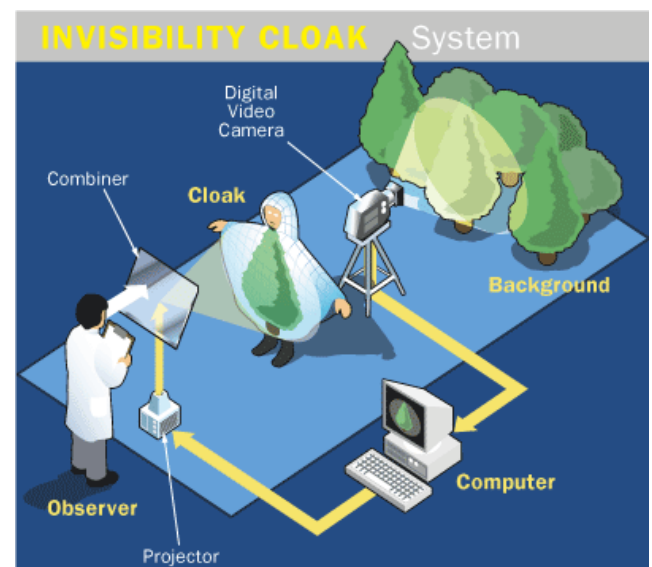


SRR act as  $LC$ -oscillators. Since the capacitance and inductance are determined by the dimensions of the SRR, scaling of the structure allows to tune the resonance frequency from microwave to terahertz to the infrared

Prerequisite for the magnetic response is the excitation of a circulating current in the individual SRR by the incident field. This induces a magnetic field which can lead to an effective negative permeability  $\mu$

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## O2-Invisibility cloak



Susumu Tachi 2003

**However: Optical Camouflage!**

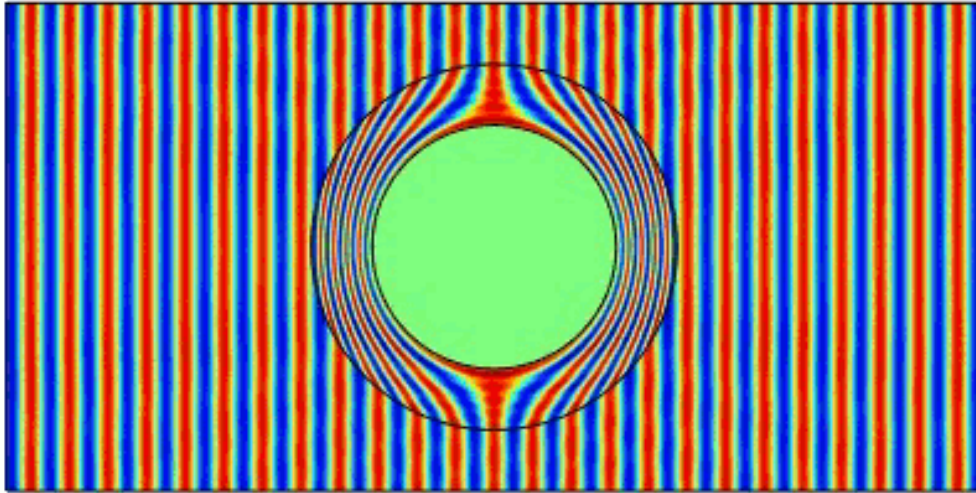
Project in front what seen in back. Not "genuine" see through

209

## Invisibility cloak (genuine!)

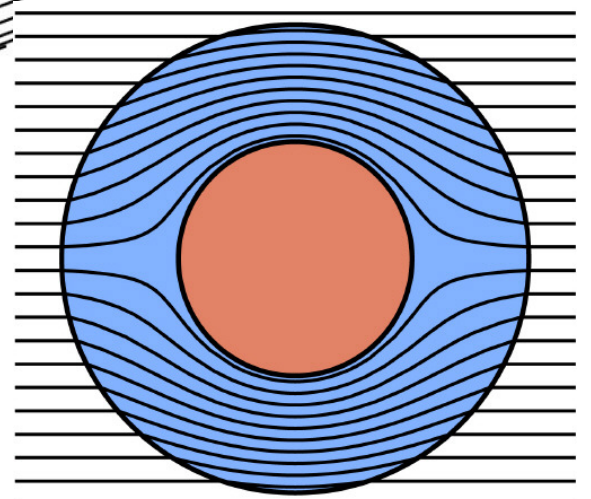
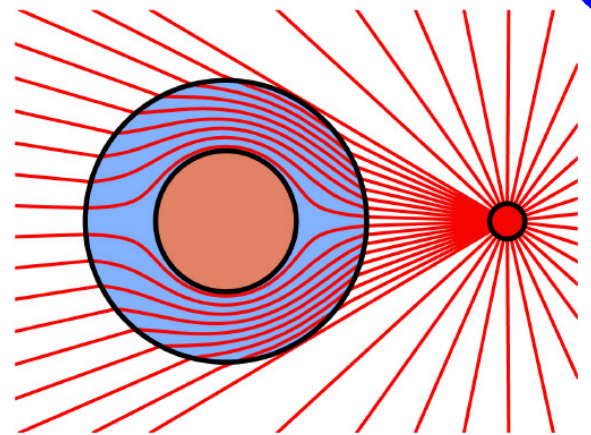
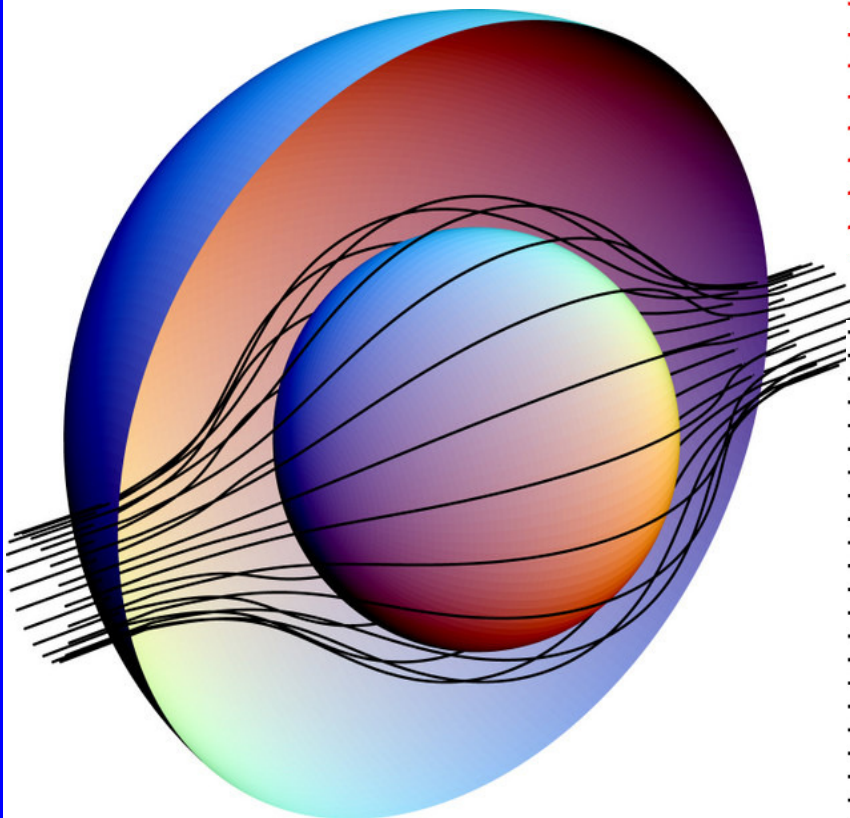
This is another example of a new optical device which could result from our ability to tailor an optical material properties

It relies on a controlled spatial variation of permittivity and permeability guiding light around the central part of the cloak



The plane wave impinging from left is "flowing" around the cloak without being disturbed by the metallic cylinder in the middle **210**

Pendry-Smith 2000 onwards





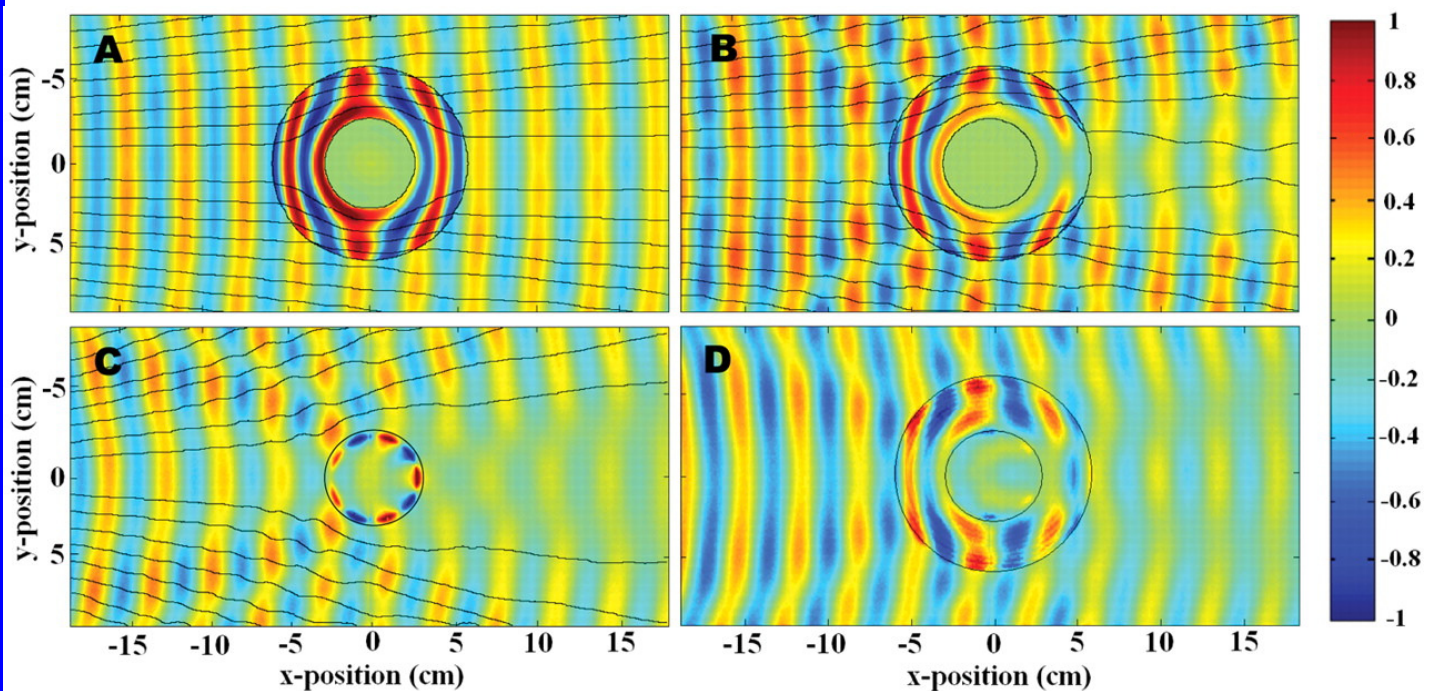
This cloaking device is practically invisible...

If you see the world in microwaves with a wavelength of 3.5cm

Schurig et al. Science 2006

212

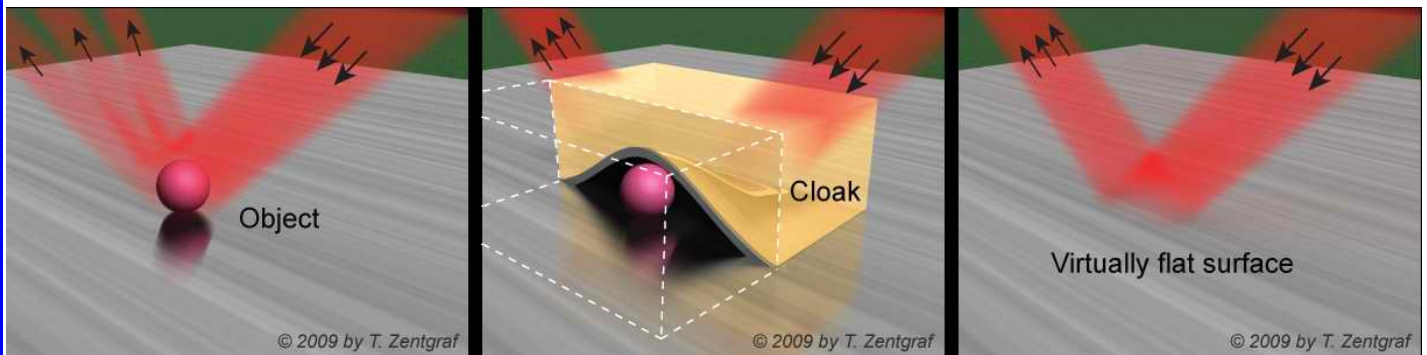
## Simulations



Experiment

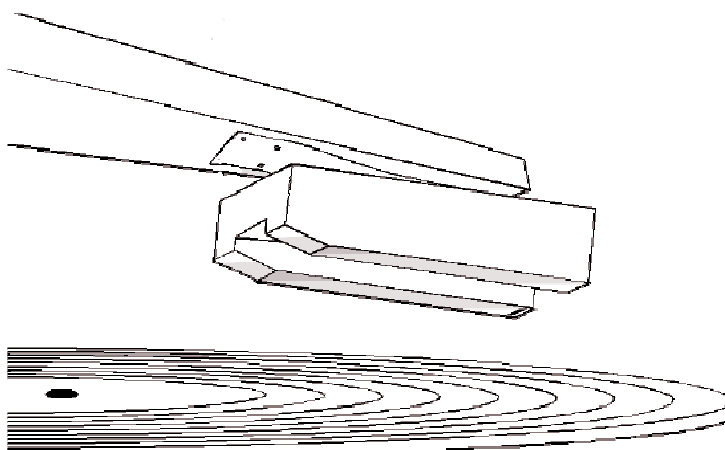
Schurig et al. Science 2006

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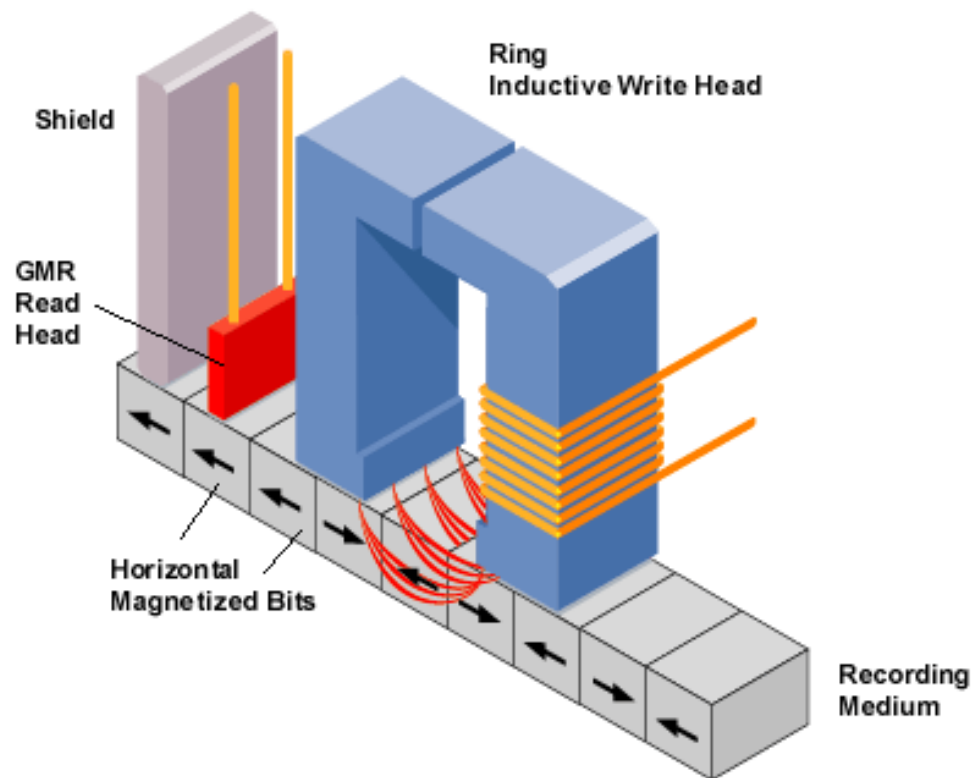
214

## O3-Hard disk drives



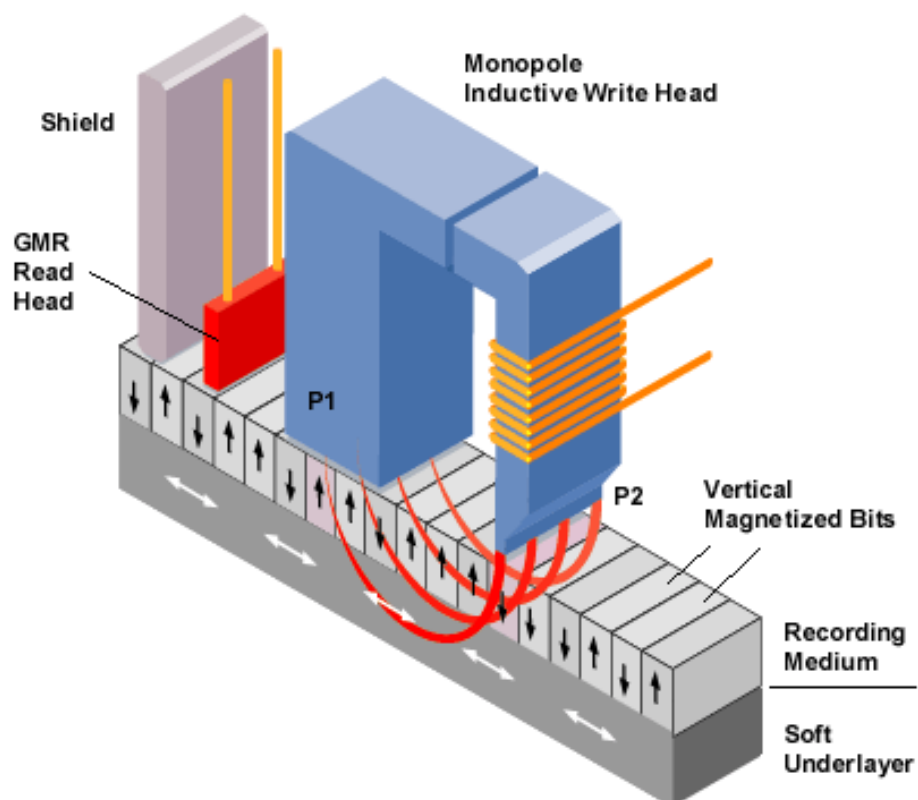
215

## Parallel Recording

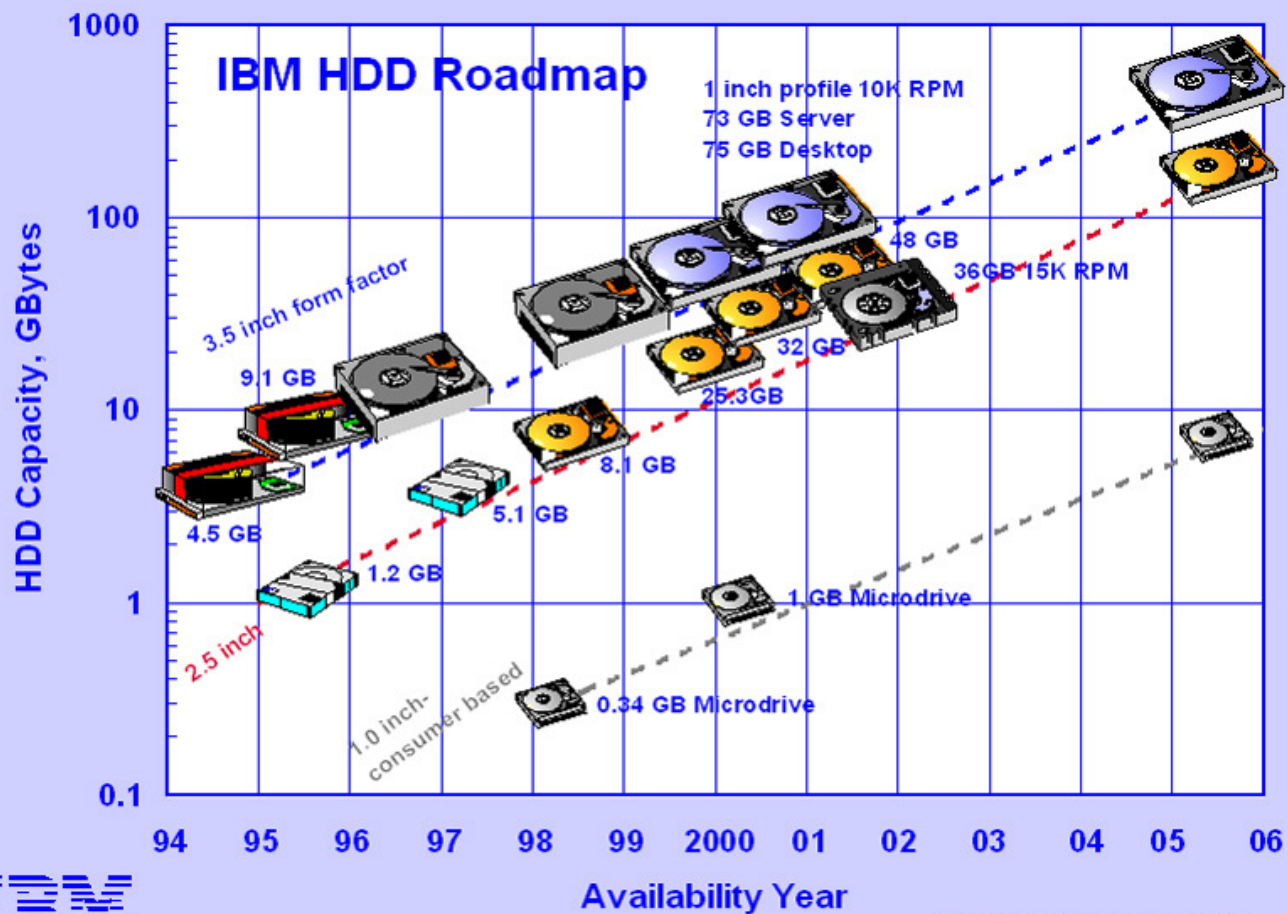


216

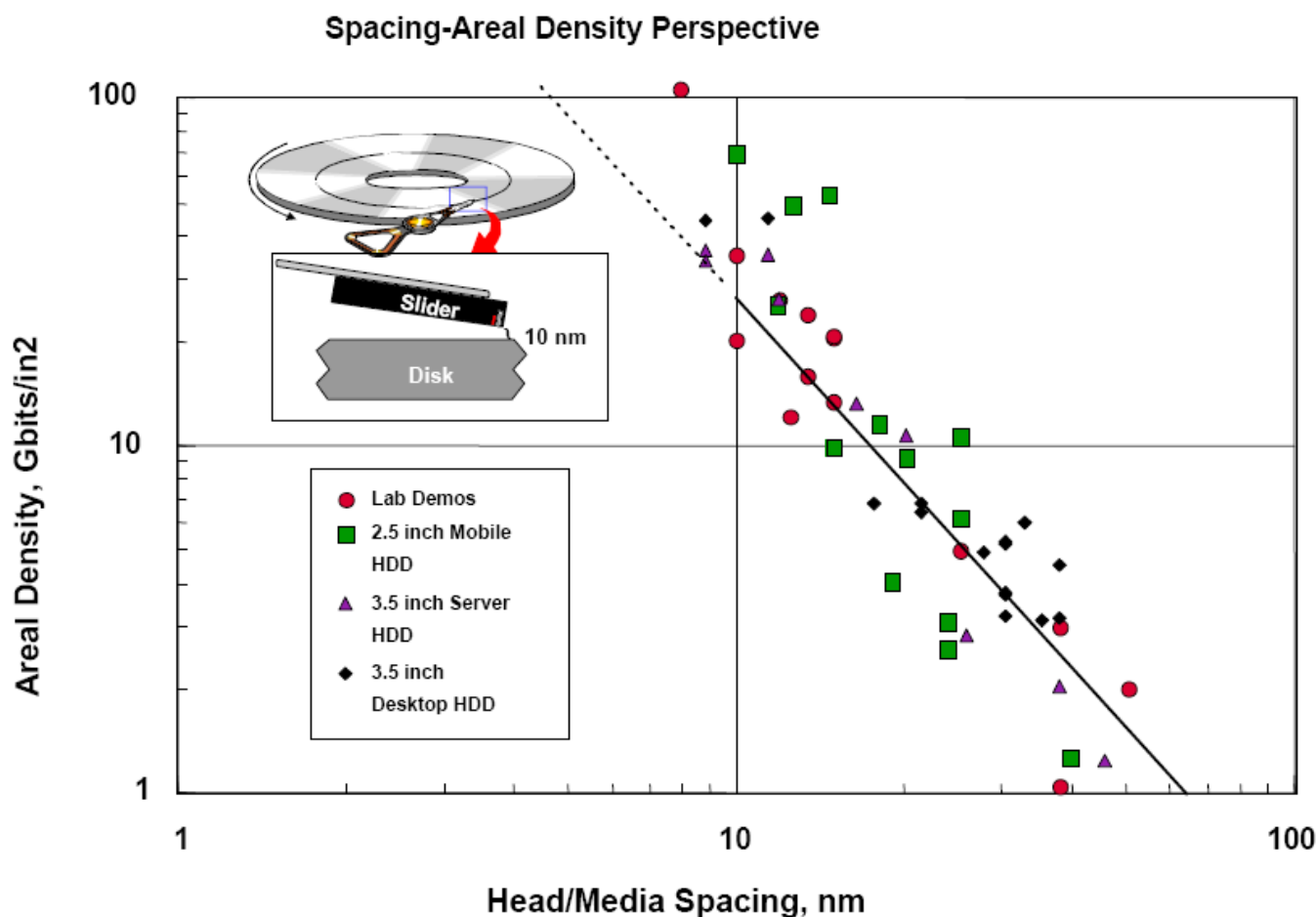
## Perpendicular Recording



217



218



219

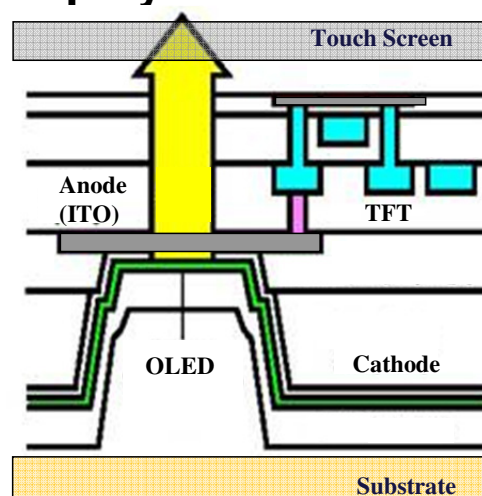
## Mini-Hard Drive



220

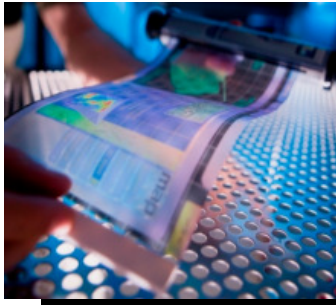
## O4-Transparent conductors

### Flexible, Foldable AMOLED Display



- Front Plane : Touch Screen, OLED
- Back plane : TFTs

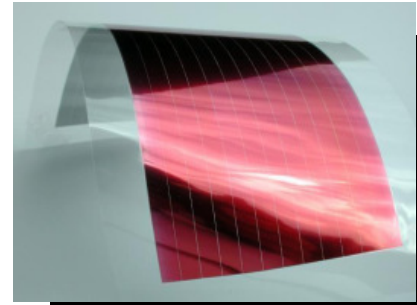
221



Touch screen displays



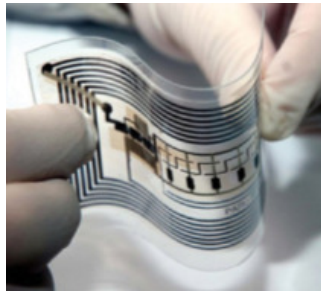
Electronic paper



Photovoltaic cells



Sensors



Radio frequency tags



Smart textile

222

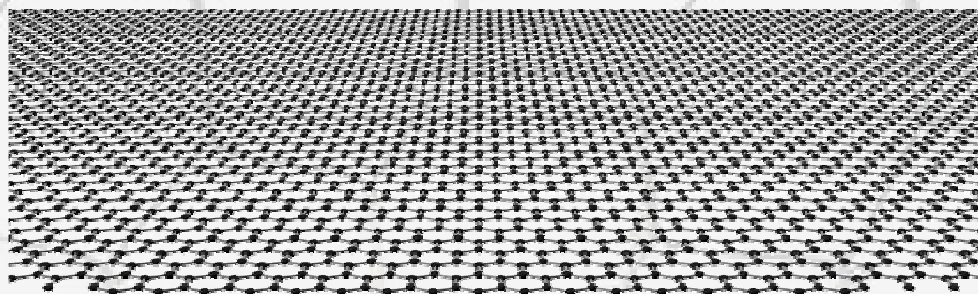
## ITO (Indium-Tin-Oxide) drawbacks

- Increasing cost due to Indium scarcity
- Processing requirements, difficulties in patterning
- Sensitivity to acidic and basic environments
- Brittleness
- Wear resistance

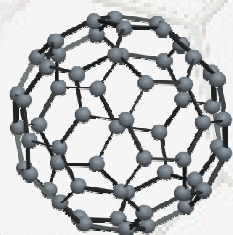
223

# Carbon Allotropes

2d  
Graphene

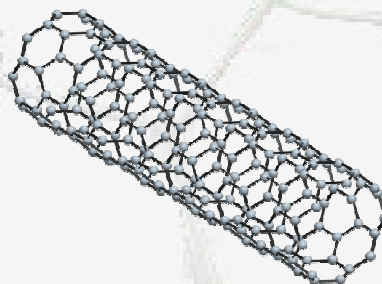


0d



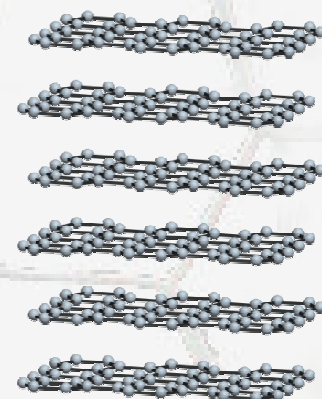
"Buckyball"

1d



Carbon Nanotube

3d

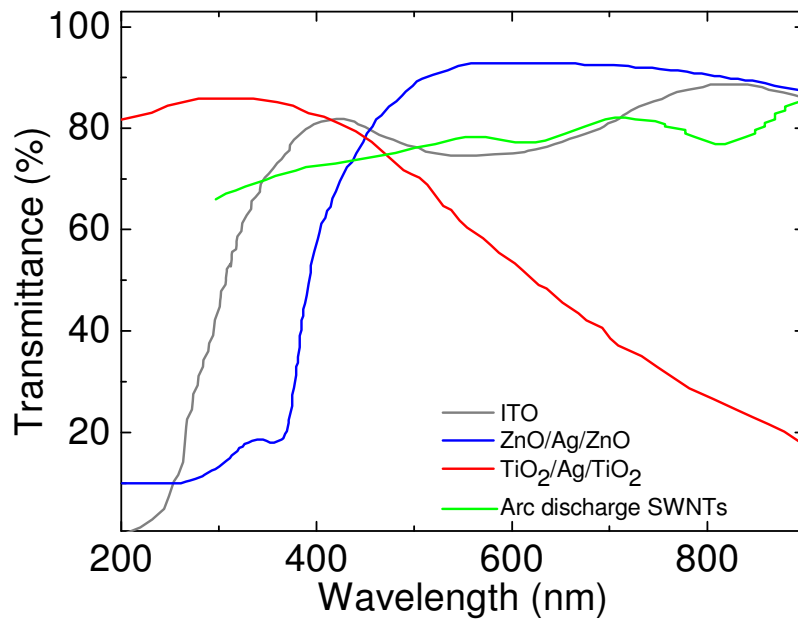


Graphite 224

## Bendability of Electronic Materials

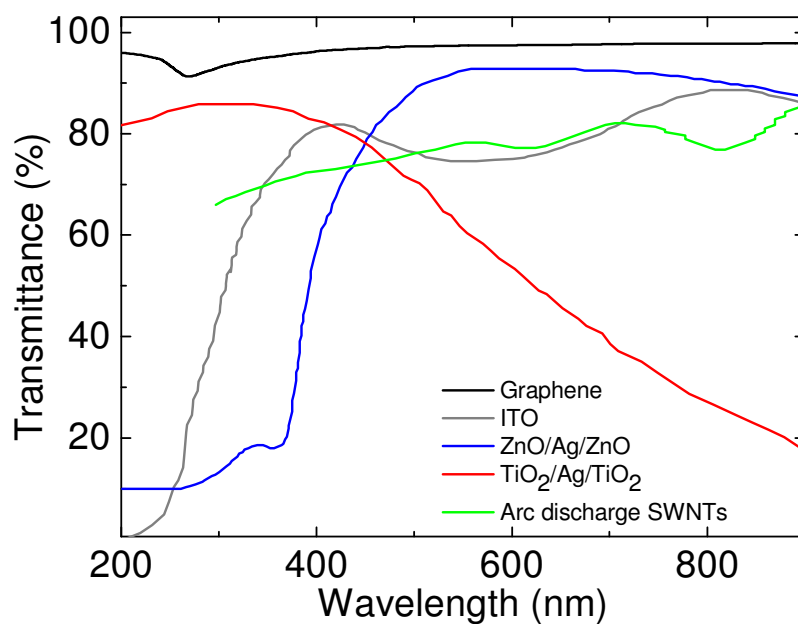
| Material | Fracture Strain | Material  | Fracture Strain |
|----------|-----------------|-----------|-----------------|
| Silicon  | ~ 0.7%          | Poly- ZnO | 0.03%           |
| ITO      | 0.58 ~ 1.15%    | Polyimide | 4%              |
| Au       | 0.46%           | Graphene  | >15-20%         |

# ITO replacements



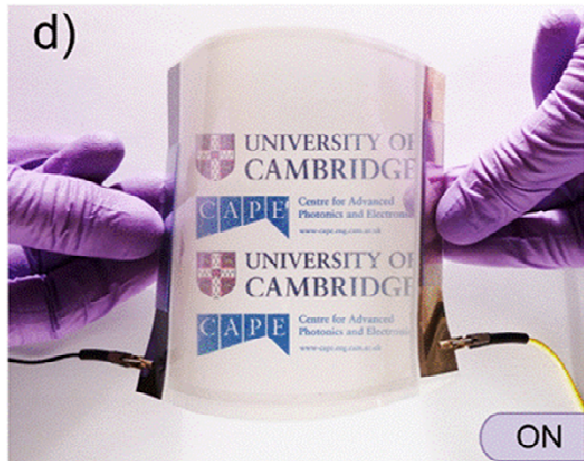
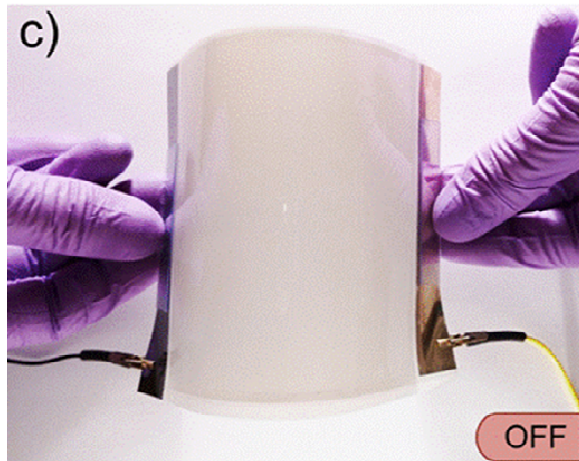
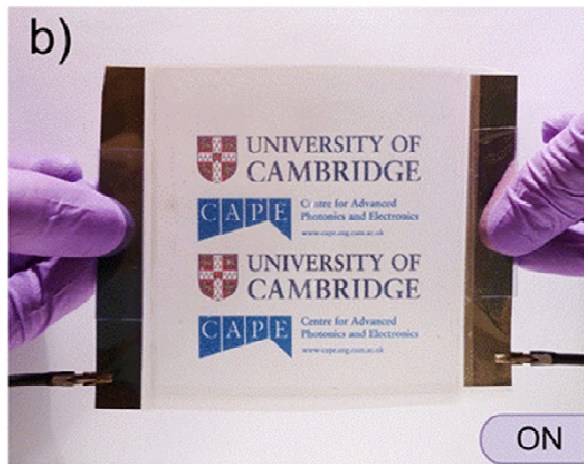
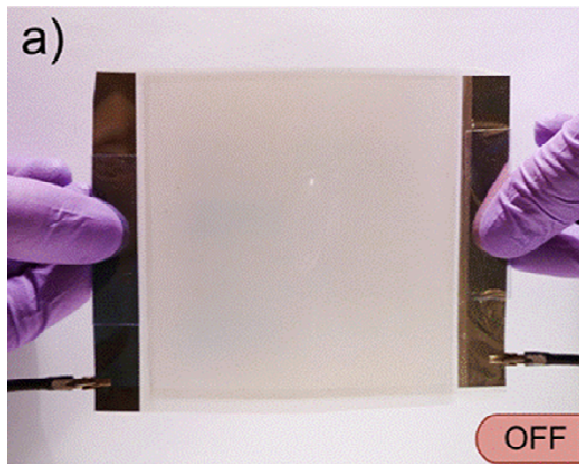
Metal grids, metallic nanowires, metal oxides and nanotubes (SWNTs) have been explored as ITO alternative

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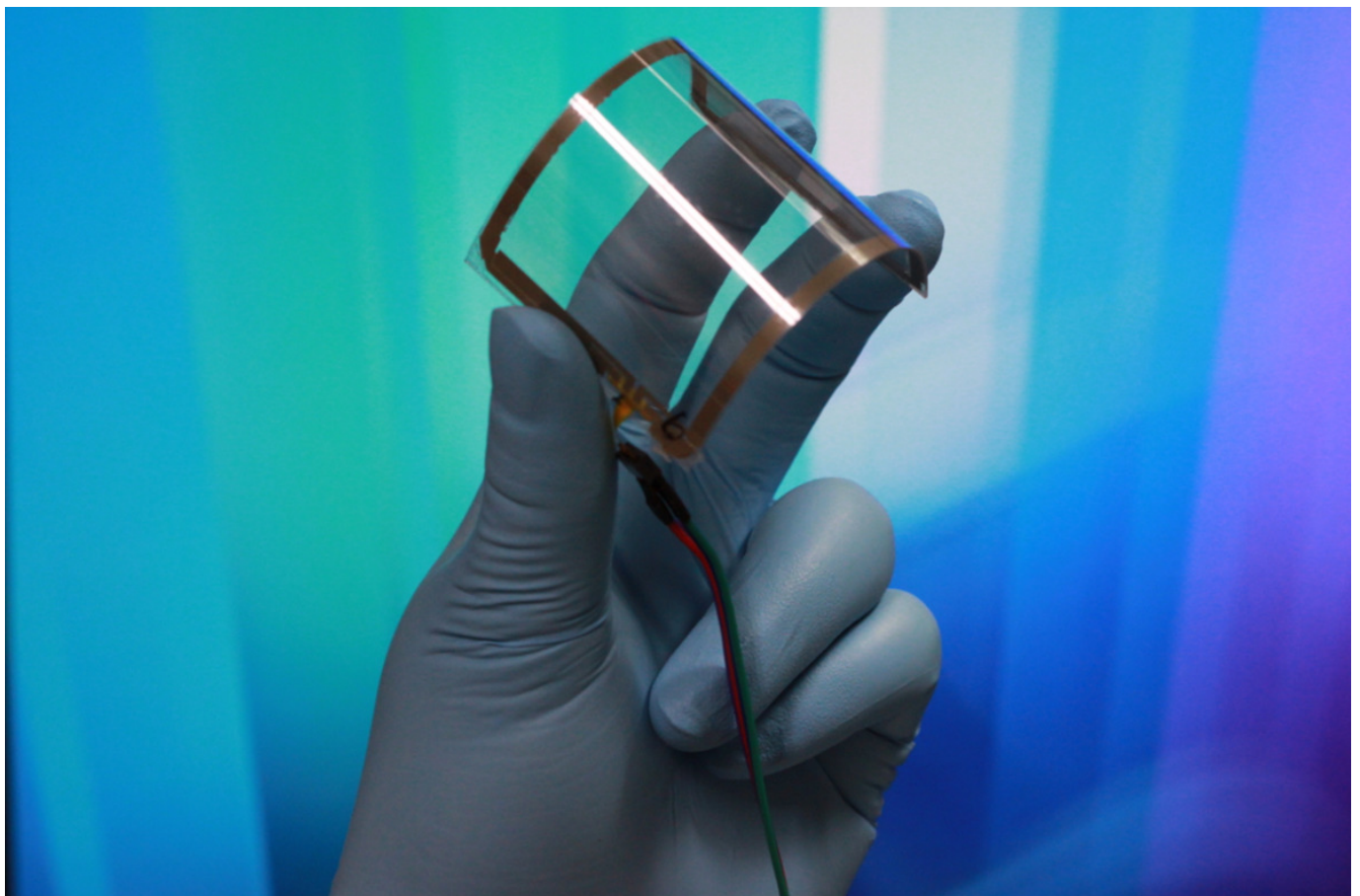


Graphene films have higher T over a wider wavelength range with respect to SWNT films, thin metallic films, and ITO

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