PART IA EXPERIMENTAL ENGINEERING

STRUCTURES RESEARCH LABORATORY MICHAELMAS TERM

THE STATICAL EQUILIBRIUM OF PLANE FRAMEWORKS

1. OBJECTIVES

(i) To investigate the load distribution within three triangulated structural frameworks both by experiment and by calculations based on the principle of statical equilibrium and the hypothesis of frictionless pin-joints between members, and

(ii) to compare some aspects of the performance of structures fabricated from welded steel tubing, extruded aluminium section and carbon fibre reinforced plastic (CFRP) .

2. THEORY

Equilibrium. If a body is in static equilibrium, then the *vector sum* of all the external forces that act upon it must be zero. This means that there will be no resultant force acting on the body in any direction we might choose to examine. In addition the *moments* of the applied forces must sum to zero about any point or any line. A diagram showing the applied forces acting on the selected component is called a *free body diagram.* Consider the application of these ideas to the combination of the lever arm and the load cell, whose free body diagram is shown in Fig. 1; if one section of the arm is three times as long as the other write down two equations to find the values of the forces Q and R in terms of the force P.

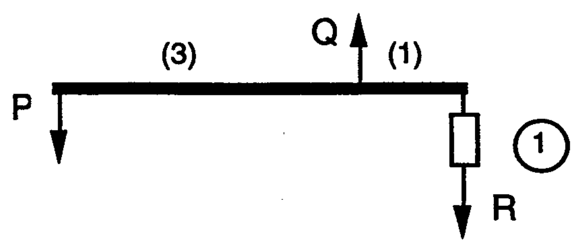


Fig. 1

R = ______P (1)

Q = ______P (2)

Pin-jointed frameworks. Triangulated frameworks are widely used in structural engineering for bridges, roof supports, derricks, off-shore oil platforms, space structures etc.; their analysis, and hence their design, can be greatly simplified by idealising the connections between individual members as frictionless hinges, or *pin-joints*. The members of a pin-jointed framework, which carries applied loads only at the joints, are either in pure tension or compression. The ideas of equilibrium apply equally well to the framework as a whole or to any part of it.

From a free body diagram of the whole framework loaded by force Q shown in Fig. 2 we can calculate the external reactions S and T. Individual member forces can then be evaluated from a series of free body diagrams for its joints; note that it is possible to use ideas of *symmetry* to simplify these calculations.

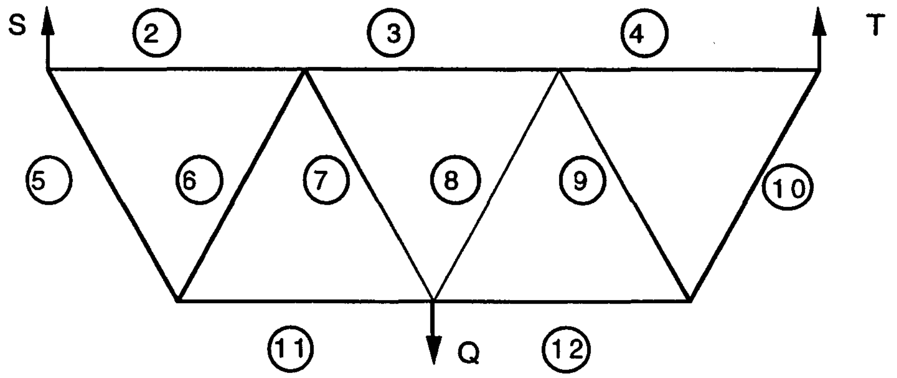


Fig. 2

3. APPARATUS

Three plane frameworks with the geometry shown in Fig. 2 are available. All the bars are the same length. The first framework is made from thin-walled steel tubes welded together at their ends; the second bolted together from sections of extruded aluminium alloy and the third constructed from lengths of carbon fibre reinforced plastic (CFRP) glued into aluminium end fittings which are then held together by steel pins. The cross-sectional areas of the steel, aluminium and CFRP members are respectively 406 mm^2, 952 mm^2 and 339 mm^2. The steel frame has a mass of 43 kg and the aluminium frame 39 kg. The CFRP frame has a mass of 17 kg of which the joints make up 11 kg and the members 6 kg. Each frame is loaded through a lever arm of the proportions of Fig. 1. A number of cast iron weights on a scale pan make up the load P.

To enable you to measure the forces in the frameworks each member, and also one of the supports, is fitted with strain gauges (labelled ② to ⑫ in Fig. 2) which measure axial strains under load: a digital display attached to each member shows the strain gauge measurement for that member. All the gauges have the same calibration factor.

In order to convert these strain readings into the required force measurements, a short length of the same material from which the frame is made (fitted with identical gauges and measuring instrumentation) is used as a *load cell* to measure the force R on the loading arm - this is shown as link ① in Fig. 1.

4. EXPERIMENTAL PROCEDURE

We start by recording the way in which the strains in the members vary with the magnitude of the load in the scale pan P, and hence with the load on the structure Q.

You will work as a team, investigating the behaviour of one of the frames, and then pool your results with those from the teams working on the other two frames. Each individual investigator will have responsibility for noting down the readings from two or three of the strain measuring bridges as the loads are applied and removed: the demonstrator will explain in more detail.

With no additional load on the scale pan note down the zero readings of your frame members from the digital displays. The load should be applied in steps of 56 lbf (i.e 0.249 kN); note down in Table I the readings from the load cell ① and the members for which you have responsibility. Continue adding weights to the scale pan to a maximum of 2.24 kN; then unload, again in steps of 0.249 kN, once more noting the values of the digital displays.

Table I - measured digital strain read-outs

Type of framework ______.

load on			digital read-outs			
scale pan, P	load cell ①,R	frame Q	load cell ①	member number		
kN	kN	kN	digits	digits	digits	digits
0						
0.249						
0.498						
0.747						
0.996						
1.245						
1.495						
1.744						
1.993						
2.242						

5. DERIVATION OF MEMBER FORCES FROM MEASURED READ-OUTS

Calibration of load cell. Plot the digital readings from the load cell ① versus the force R. Hence, from the slope of the curve, obtain the calibration factor of the load cell in terms of digits/kN of applied load.

Load cell calibration factor _____digits/kN (3)

Member forces. Plot the values of the digital read-out for your particular bars in the framework (which you have noted in Table I) versus the load Q on the framework. From these curves estimate the digital read-outs you would expect to be induced by a load Q = 5 kN and also Q = 0. Note these values in Table II. By looking at the *differences* between these figures, and using the calibration factor of equation (3), you will be able to estimate the *force* in each member when Q = 5 kN. Note these values in the appropriate column of Table II for 'your' particular bars.

Table II - Digital read-outs and member forces for a load Q = 5 kN

Type of framework ______________.

	digital read-outs			
member	Q = 5 kN estimate (a)	Q = 0 "no load" (b)	difference (a) - (b)	estimated member force (kN) for Q = 5 kN

Confer with the other members of your team and use their member forces, as well as your own, to complete the relevant column of Table III.

Table III - Estimated member forces for a load Q = 5 kN

member	steel framework	Al-alloy framework	CFRP framework
load cell ①			
②			
③			
④			
⑤			
⑥			
⑦			
⑧			
⑨			
⑩			
⑪			
⑫			

Comparison of the three frameworks. At a meeting of the whole class, i.e. all three teams, complete Tables III and IV.

Table IV - Load cell calibration factors (digits/kN)

Steel framework	Al-alloy framework	CFRP framework

6. THEORETICAL CALCULATION OF MEMBER FORCES

What member forces would you expect to be generated by a load Q = 5 kN on a pin-jointed framework of the given geometry? Complete Table V with these values which can be found by considering the equilibrium of each of the pin-joints. Use the convention + for tension and - for compressive forces.

Table V: expected member loads for a load Q = 5 kN

member	expected load/kN
load cell ①	
②	
③	
④	
⑤	
⑥	
⑦	
⑧	
⑨	
⑩	
⑪	
⑫	

7. REPORT

General guidance on report writing can be found in the IA document 'A Guide to Report Writing'. For this particular investigation, your report, which you will write in the booklet provided, should contain the following:

1. The *Title* page including your name, college and group number. The *Summary*, also on the title page, should contain a very brief resumé of what you have done, why you have done it and what you have concluded - all in not much more than 100 words.

2. An *Introduction and Objectives* in which you briefly explain the background to the work to be described and justify why the investigation is worth carrying out. Your statement of the objectives of the experiment can be based on those given in Section 1 of this document.

3. There is no need to repeat all the details of the *Apparatus* and *Experimental Method* given in this handout. However, you should give a brief account of the *Theory* that you used to calculate the values of bar forces in Table V.

4. The best way to present numerical data in the section on *Results* is in tabular form - think out the form of the table it would be best to use: tables, like diagrams, should be numbered and have titles. The raw data (i.e. the numbers you actually read off the equipment and collected in Table II) can go into the Appendix. Numerical information, such as the calibration data, is often best displayed graphically.

5. It is good practice to separate the *presentation* of results (see para. 4 above) from their *Discussion*: this section should include a comparison between what you might expect to observe and what you actually measured. Any significant variations should be the subject of comment. You should give some thought to the following specific points which you may like to discuss with your structures supervisor:

(i) The three frames are made of very different materials joined together in very different ways. What influence has this had on the distribution of forces within the frameworks? What can you conclude about pin-jointed analysis? Would you expect the same conclusion to hold if the frameworks were made of much stockier members?

(ii) Consider the equilibrium of a joint of the structure which involves one of the members whose load you have been monitoring. For the load case when Q = 5 kN (i.e. the data in Table III) draw a free body diagram for the joint and the corresponding force polygon. Repeat for at least one other joint. Do the polygons close - if not, can you explain why not?

(iii) The strain gauges measure the extension or contraction per unit length of the members to which they are attached. What feature of the material behaviour leads to our being able to describe the calibration of the load cell by a single value of digits/kN? The digital displays are such that a change in the digital reading of 100 corresponds to a strain of 35.3×10^{-6} (which is sometimes written as 35.3 micro-strain). Cross-sectional areas of the members are given in §3; estimate the elastic moduli of the three materials.

(iv) Why was it possible to neglect the weight of the framework, the scale pan and the loading arm in the calculations you have undertaken?

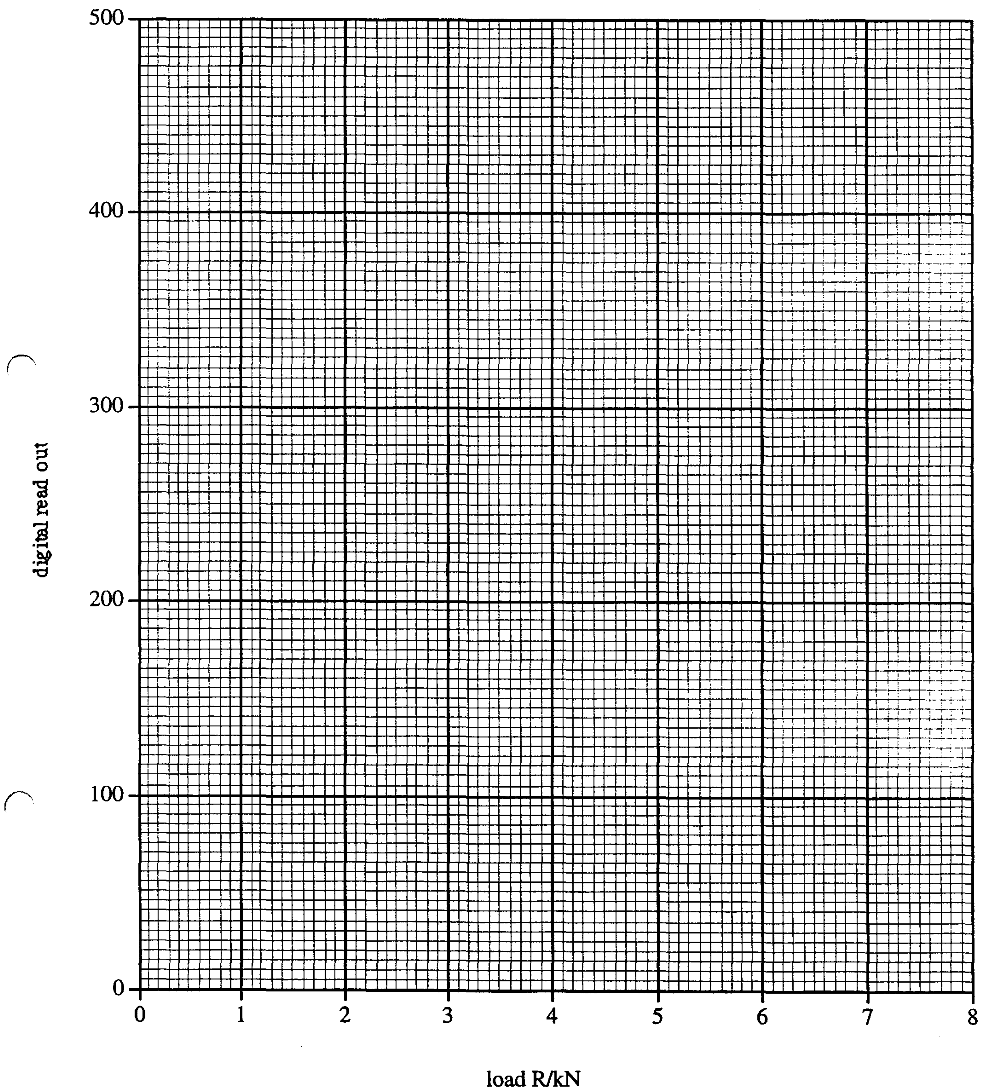
(v) The mass of material used in each of the three frames is given in the table below. Compare the frames by defining an index of performance as specific stiffness (i.e. the applied load per unit deflection of the frame at the loaded point per unit mass of frame). You might note that in production the mass of material used in the joints of the CFRP frame could probably be reduced by about 50%.

Data are also provided on the costs of the frames (both material and fabrication). What do you conclude from these figures?

frame	Steel	Al - alloy	CFRP
mass (kg)	43	39	17 total, joints 11
<u>costs</u> material fabrication	 £25 £800	 £145 £600	 £785 £2000
total	£825	£745	£2785

6. *Conclusions*; these should summarise concisely the basic achievements of the investigation in the light of the aims and objectives; it is often a good idea to number the conclusions.

JML/JAW
September 02


500
400
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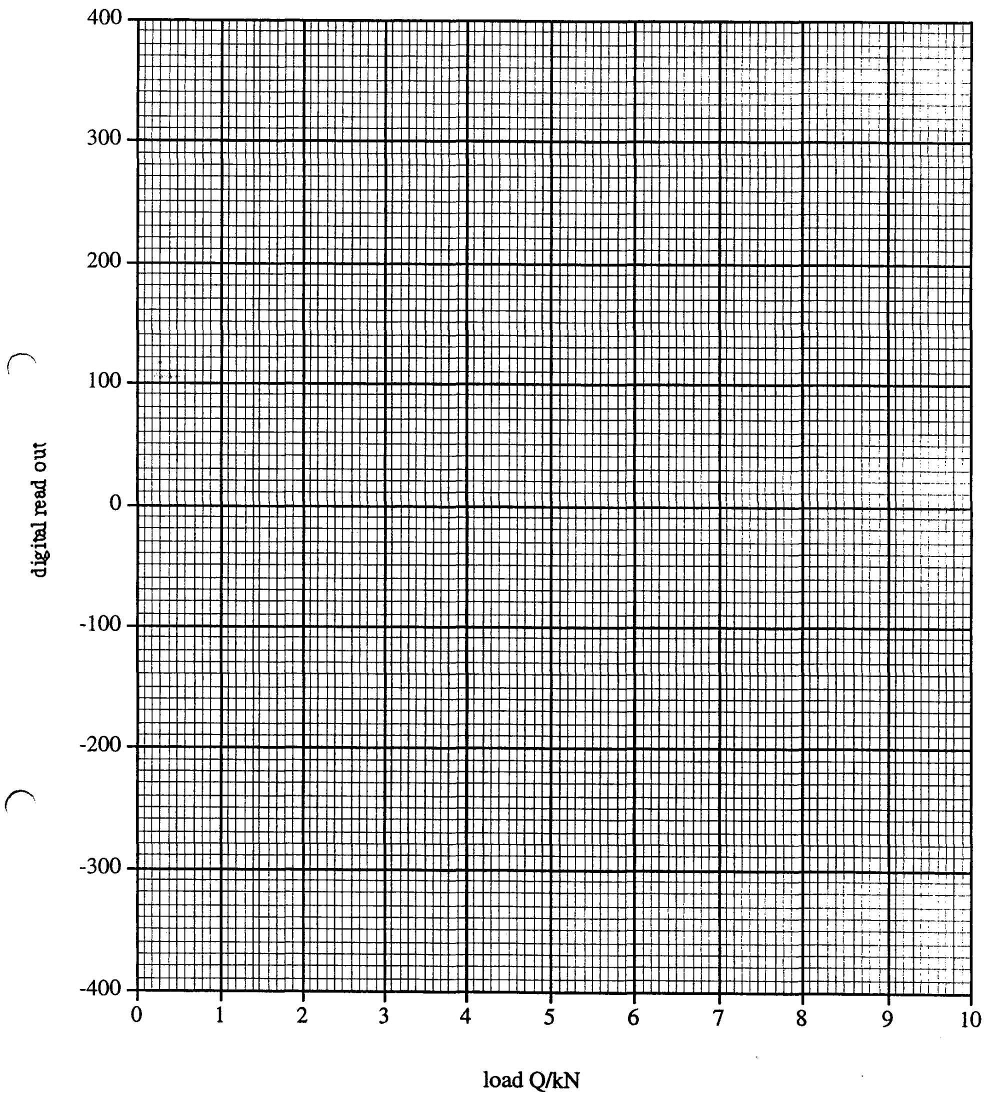
100
0
0 1 2 3 4 5 6 7 8
digital read out
load R/kN



digital read out
400
300
200
100
0
-100
-200
-300
-400
0
1
2
3
4
5
6
7
8
9
10
load Q/kN