PART IA EXPERIMENTAL ENGINEERING

STRUCTURES RESEARCH LABORATORY MICHAELMAS TERM

THE STATICAL EQUILIBRIUM OF PLANE FRAMEWORKS

On the day of your lab, please note down the following:

Date of lab ☐ Session password ☐

You will need this information if you wish to access your lab data electronically - see:
http://www-h.eng.cam.ac.uk/help/tpl/php/1AStatics/index.php

1. OBJECTIVES

(i) To investigate the load distribution within three triangulated structural frameworks both by experiment and by calculations based on the principle of statical equilibrium and the hypothesis of frictionless pin-joints between members, and

(ii) to compare some aspects of the performance of structures fabricated from welded steel tubing, extruded aluminium section and carbon fibre reinforced plastic (CFRP) .

2. THEORY

Equilibrium. If a body is in static equilibrium, then the *vector sum* of all the external forces that act upon it must be zero. This means that there will be no resultant force acting on the body in any direction we might choose to examine. In addition the *moments* of the applied forces must sum to zero about any point or any line. A diagram showing the applied forces acting on the selected component is called a *free body diagram*. Consider the application of these ideas to the combination of the lever arm and the load cell, whose free body diagram is shown in Fig. 1; if one section of the arm is three times as long as the other write down two equations to find the values of the forces Q and R in terms of the force P.

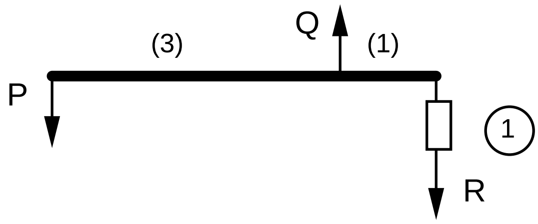

Fig. 1

R = ______P (1)

Q = ______P (2)

Pin-jointed frameworks. Triangulated frameworks are widely used in structural engineering for bridges, roof supports, derricks, off-shore oil platforms, space structures etc.; their analysis, and hence their design, can be greatly simplified by idealising the connections between individual members as frictionless hinges, or *pin-joints*. In a pin-jointed framework with straight members, which carries applied loads only at the joints, the members are either in pure tension or compression. The ideas of equilibrium apply equally well to the framework as a whole or to any part of it.

From a free body diagram of the whole framework loaded by force Q shown in Fig. 2 we can calculate the external reactions S and T.

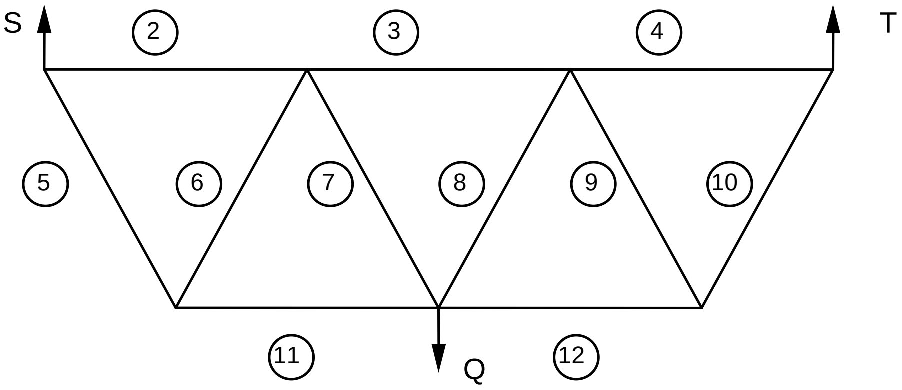

Fig. 2

S = (3)

Individual member forces can then be evaluated from a series of free body diagrams for its joints; note that it is possible to use ideas of *symmetry* to simplify these calculations. A free body diagram for joint A is shown in Fig. 3. This can be used to find the bar forces T_2 and T_5 in terms of S.

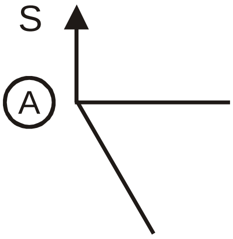

Fig. 3

T_5 = (4)

T_2 = (5)

3. APPARATUS

Three plane frameworks with the geometry shown in Fig. 2 are available. All the bars are the same length. The first framework is made from thin-walled steel tubes welded together at their ends; the second bolted together from sections of extruded aluminium alloy and the third constructed from lengths of carbon fibre reinforced plastic (CFRP) glued into aluminium end fittings which are then held together by steel pins. The cross-sectional areas of the steel, aluminium and CFRP members are respectively 406 mm^2, 952 mm^2 and 339 mm^2. The steel frame has a mass of 43 kg and the aluminium frame 39 kg. The CFRP frame has a mass of 17 kg of which the joints make up 11 kg and the members 6 kg. Each frame is loaded through a lever arm of the proportions of Fig. 1. A number of cast iron weights on a scale pan make up the load P.

To enable you to measure the forces in the frameworks each member is fitted with strain gauges which consist of several strain gauges wired together to measure axial strain under load (labelled ② to ⑫ in Fig. 2): a digital display attached to each member shows the strain gauge measurement for that member. All the gauges have the same calibration factor.

In order to convert these strain readings into the required force measurements, a short length of the same material from which the frame is made (fitted with identical gauges and measuring instrumentation) is used as a *load cell* to measure the force R on the loading arm - this is shown as member ① in Fig. 1. From equilibrium we know the force R for any given load in the scale pan P (eqn 1). In a similar manner, the load on the structure Q for a given load P can also be calculated (eqn 2). The loading increments to be considered in the experiment are shown in Table I. Calculate the corresponding values of R and Q for each load step.

4. EXPERIMENTAL PROCEDURE AND DERIVATION OF MEMBER FORCES FROM MEASURED READ-OUTS

We start by recording the way in which the strains in the members vary with the magnitude of the load in the scale pan P, and hence with the load on the structure Q. Of particular interest will be to determine the bar forces in all three frames when Q = 5 kN.

Each frame will be loaded in three phases; phase 1 (loading from P = 0 to 0.996 kN), phase 2 (loading from P = 1.245 to 2.242 kN) and phase 3 (unloading). You will work as a team and each team will move to a different frame after each loading phase. Your team is therefore responsible for investigating the behaviour of each frame during a particular loading phase, and then pooling your results with those from the teams who have noted the behaviour during the other loading phases. Each individual investigator will have responsibility for the readings from three bars (the same three bars on all three frames) by noting down the readings from the strain measuring bridges: the demonstrator will explain in more detail.

PHASE 1

In Table I, note the framework you are investigating. With no additional load on the scale pan note down the zero readings of your frame members from the digital displays. The load should be applied in steps of 56 lbf (i.e 0.249 kN); note down in Table I the readings from the load cell ① and the members for which you have responsibility. Continue adding weights to the scale pan up to 0.996 kN. As a team, enter the readings from each team member into the computer associated with your frame.

Eqn 1 was used to find the force in the load cell, R, corresponding to an applied load P at each load step. Hence it is possible to plot the digital readings from the load cell ① versus the force in the load cell R (kN) for your frame during phase 1 loading. From the slope of the curve, the calibration factor of the load cell in terms of digits/kN of applied load can be determined. By combining the results from all three teams, the phase 1 calibration factors for the steel, aluminium and CFRP frames can be entered in Table II.

Using the appropriate calibration factor and a knowledge of the member digital readings at Q = 0 kN and Q = 3.984 kN, it is possible to predict the member forces at a load Q = 5 kN. Your demonstrator will work through an example for members 2 and 5 (see Table III).

PHASE 2

Move to the next frame and note the type of framework in Table I. Add weights to the scale pan in steps of 56 lf (0.249 kN) up to a load of 2.242 kN. Again note the digital readings for the load cell and the members for which you have responsibility - your first reading will be at a load of 1.245 kN. As a team, enter the readings from each team member into the computer associated with your frame.

It is now possible to make a plot of the digital readings from the load cell ① versus the force in the load cell R for each of the frames for both phase 1 and 2 loading. From the slope of the curve, an updated calibration factor of the load cell in terms of digits/kN of applied load can be determined. Note the combined phase 1 and 2 calibration factors in Table II.

PHASE 3

Move to the next frame. Remove the weights from the scale pan in steps of 168 lf (0.747 kN) down to a load of 0 kN. At each step, note the readings for the load cell and the members from which you have responsibility. As a team, enter the readings for each team member into the computer associated with your frame.

The plot of the digital readings from the load cell ① versus the force in the load cell R for each of the frames during both loading and unloading can now be plotted.

Table I - measured digital strain read-outs

PHASE 1			Type of framework __________			
load on			digital read-outs			
scale pan, P	load cell ① , R	frame Q	load cell	member number		
			①			
kN	kN	kN	digits	digits	digits	digits
0		0				
0.249						
0.498						
0.747						
0.996		3.984				
PHASE 2			Type of framework __________			
load on			digital read-outs			
scale pan, P	load cell ① , R	frame Q	load cell	member number		
			①			
kN	kN	kN	digits	digits	digits	digits
1.245						
1.495						
1.744						
1.993						
2.242						
PHASE 3			Type of framework __________			
load on			digital read-outs			
scale pan, P	load cell ① , R	frame Q	load cell	member number		
			①			
kN	kN	kN	digits	digits	digits	digits
2.242						
1.495						
0.747						
0						

Table II – Experimental load cell calibration factors (digits/kN) – for loading

	Steel framework	Al-alloy framework	CFRP framework
Phase 1 only			
Phase 1 and 2 (overall)			

QUESTION 1 – Why are the calibration factors different for the different frames?

Table III - Digital read-outs and experimentally predicted member forces for bars 2 and 5 at load Q = 5 kN – Sample calculation after Phase 1

		Digital readouts				
Frame	Member	Q = 3.984 kN (a)	Q = 0 kN (no load) (b)	Difference (a)-(b)	Estimated digit difference for Q = 5 kN	Estimated member force for Q = 5 kN
Steel	②					
	⑤					
Aluminium	②					
	⑤					
Carbon	②					
	⑤					

QUESTION 2 – Is there any similarity between the estimated experimental member forces for the three frames? If so, what does this imply?

QUESTION 3 - Are there any pitfalls in relying on only two digital readings (at Q = 3.984 kN and Q = 0 kN) in making your estimation?

5. ESTIMATED EXPERIMENTAL MEMBER FORCES FOR Q=5 kN

By combining your team's experimental results with those of the other teams, it is possible to estimate each member force at a load of Q = 5 kN (see Table IV). The computer is used to calculate the overall load cell calibration factor for the relevant material (best fit over phase 1 and 2 loading) and a best fit line of digits versus Q for each member (between Q = 8.968 kN and Q = 0 kN). An experimental estimate of the bar force in each member at Q = 5 kN can then be obtained.

QUESTION 4 – Is it better to use the Phase 1 load cell calibration or the combined Phase 1 and 2 load cell calibration factor?

Table IV - Estimated experimental member forces for a load Q = 5 kN

Member	Steel framework	Al-alloy framework	CFRP framework
Load cell ①			
②			
③			
④			
⑤			
⑥			
⑦			
⑧			
⑨			
⑩			
⑪			
⑫			

QUESTION 4 – Are there any quick checks that can be made to spot inconsistencies in the results?

6. FORCE POLYGONS

Draw the experimental force polygons for joint A for all three frames using the relevant readings from Table IV.

	Free body	Force polygon	Resultant force
Steel			
Aluminium			
CFRP			

QUESTION 5 – Should the force polygons close? If the force polygons do not close then why is this?

7. PIN-JOINTED CALCULATION OF MEMBER FORCES

What member forces would you expect to be generated by a load Q = 5 kN on a pin-jointed framework of the given geometry? Complete Table V with these values which can be found by considering the equilibrium of each of the pin-joints. Use the convention + ve for tension and – ve for compressive forces.

For example, for Q = 5 kN, S= and pin-jointed analysis gives , T_5 =
and T_2 = (equations 4 and 5).

Table V: Pin-jointed calculation of member forces for a load Q = 5 kN

Member	Expected load (kN)
Load cell ①	
②	
③	
④	
⑤	
⑥	
⑦	
⑧	
⑨	
⑩	
⑪	
⑫	

QUESTION 6 – How do these values compare with the estimates based on the experimental readings (see Table IV)?

8. REPORT

General guidance on report writing can be found in the IA document ‘A Guide to Report Writing’. For this particular investigation, your report, which you will attach to the coversheet provided, should contain the following:

1. The *Title* page including your name, college and group number. The *Summary*, also on the title page, should contain a very brief resumé of what you have done, why you have done it and what you have concluded - all in not much more than 100 words.

2. An *Introduction and Objectives* in which you briefly explain the background to the work to be described and justify why the investigation is worth carrying out. Your statement of the objectives of the experiment can be based on those given in Section 1 of this document.

3. There is no need to repeat all the details of the *Apparatus* and *Experimental Method* given in this handout. Instead it is preferable to attach this handout to your report and label it as an Appendix. You can then present a very brief overview of the experiment in the main report and refer a reader to the Appendix for further details. However, you should give a brief account of the *Theory* that you used to calculate the values of bar forces in Table V.

4. Think about the best way to present numerical data in the section on *Results*. Decide on what is the clearest way to present particular data e.g. in tabular or graphical form. Think out the form of the table or graph or diagram that it would be best to use: tables and graphs, like diagrams, should be numbered and have titles. Note that numerical information, such as the calibration data, is often best displayed graphically. The raw data (i.e. the numbers you actually read off the equipment and collected in Table I) can go into the Appendix. The complete

lab results can be downloaded from the webpage noted on pg. 1 of this handout (you will need your lab date and session password to log in).

5. It is good practice to separate the *Presentation* of results (see para. 4 above) from their *Discussion*: this section should include a comparison between what you might expect to observe and what you actually measured. Any significant variations should be the subject of comment. You should give some thought to the following specific points which you may like to discuss with your structures supervisor:

(i) The three frames are made of very different materials joined together in very different ways. What influence has this had on the distribution of forces within the frameworks? What can you conclude about pin-jointed analysis? Would you expect the same conclusion to hold if the frameworks were made of much stockier members? What if the members were not straight?

(ii) Consider the equilibrium of a joint of the structure which involves one of the members whose load you have been monitoring (but *not* joint A). For the load case when Q = 5 kN (e.g. the data in Table IV) draw a free body diagram for the joint and the corresponding force polygons for each of the three frames. Do the polygons close - if not, can you explain why not?

(iii) Using your phase 1 digital readings, and your phase 1 load cell calibration factor, predict the bar forces in your members for a load of Q = 5 kN. Do your experimental bar force predictions differ from the final results compiled in Table IV? Which do you feel are more accurate?

(iv) The strain gauges measure the extension or contraction per unit length of the members to which they are attached. What feature of the material behaviour leads to our being able to describe the calibration of the load cell by a single value of digits/kN? The digital displays are such that a change in the digital reading of 100 corresponds to a strain of 35.3×10^{-6} (which is sometimes written as 35.3 micro-strain). The cross-sectional areas, A, of the members are given in §3; estimate the elastic moduli, E, of the three materials.

(v) Why was it possible to neglect the weight of the framework, the scale pan and the loading arm in the calculations you have undertaken?

(vi) The mass of material used in each of the three frames is given in the table below. Compare the frames by defining an index of performance as specific stiffness λ where $\lambda=(E\times A)/\text{mass}$. You might note that in production the mass of material used in the joints of the CFRP frame could probably be reduced by about 50%.

Data are also provided on the costs of the frames (both material and fabrication). What do you conclude from these figures?

frame	Steel	Al-alloy	CFRP
mass (kg)	43	39	17 total, joints 11
costs			
material	£25	£145	£785
fabrication	£800	£600	£2000
total	£825	£745	£2785

6. *Conclusions*; these should summarise concisely the basic achievements of the investigation in the light of the aims and objectives; it is often a good idea to number the conclusions.

JML September 05