Investigation of a double-layer tensegrity glazing system

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Summary
Tensegrity grids, with tensed cables and isolated struts, have strong aesthetic appeal, but have had limited architectural engineering applications. The present study investigates a promising application: the use of a tensegrity grid as supporting structural system for double-skin glazing systems.

The system considered consists of two layers of glass panels sandwiching a core modular tensegrity grid. The tensegrity grid consists of steel compression tubes and steel tension rods designed to pre-stress the glass supporting system. The use of structural glass is central to the design as the glass plays a double role in the structural system. Apart from their functional purpose (thermal insulation, visual transparency etc.), the glass panels also serve structural purposes acting as load bearing structural elements integrated with the tensegrity supporting system.

The paper considers the advantages of a tensegrity system, both from a functional and structural perspective, as well as considering aspects of pre-stressing and structural glass behaviour.

Keywords: tensegrity; pre-stress; grid; structural glass; double-skin glazing.

1. Introduction

Could tensegrity structures become a part of the construction industry, and a sensible technical and aesthetic choice? Tensegrity structures are fascinating, and favoured by architects and artists, but their current application in engineering is rather limited. Their singular morphology, complex geometry, constructability and limited structural redundancy have suppressed their use. However, recent investigations have shown that tensegrity systems have the potential to serve in a number of engineering applications, such as solar collector space grids and pedestrian bridges [1-2].

The present paper investigates the potential of double-layer tensegrity grids to act as structural façade enclosures through a specific case study. The novelty of such an application is further enhanced by using a ‘brittle’ material, glass, as a structural component in to the system. The application proposed is the use of glass panels as structural components forming the top and bottom layers of a double-layer tensegrity grid in order to create a double-skin glass façade with an interesting aesthetic quality to partially address the question posed in the first sentence. The application is described through a case study where a double-skin glass façade wall spans a large area, for instance the face of an atrium.
2. Double-skin tensegrity glass façade proposal

2.1 The glass panelling concept

Advances in glass façade technology have shown that the double-skin glass facades present improved thermal and acoustic properties compared with the conventional single skin glass facades [3]. However, the need for more efficient structural enclosures for double-skin glass facades remains unsatisfied, since the existing ‘open’ systems require the use of large profile anchorage structures to counterbalance the pre-stressing forces. In this paper we explore the use of an inverse bidirectional tensegrity [4] grid (Figure 1-a) acting as a ‘closed’ structural system. The idea was based on the concept suggested by Expedition Engineering and analysed in [1]. For the sake of structural efficiency and enhanced visual transparency, the top and bottom layers of strut elements from the tensegrity grid are replaced by glass panels, which act as structural components (Figure 1-b). Recent advances in structural glass have shown a great potential of using glass as a structural material with commendable structural properties [5]. Thus, the glass panels will play a double role in this study acting both as a skin, and as a structural component resisting compressive forces resulting from the applied pre-stress in the structure.

![Figure 1](image1.png)

*Figure 1: Representation of a 2×2 module bidirectional tensegrity basic unit (a) and a bidirectional tensegrity grid with top and bottom glass panel layers (b).*

2.2 The double-skin tensegrity façade wall

The double-layer façade wall investigated is a 16m×12m structure aiming to serve as the glazing face of an atrium in a tall building or the facade of an airport lounge. The proposed case study considers an 8×6 module double-layer grid glass façade composed of 2m×2m×1m modules accommodating 2m×2m square glass panels as shown in Figure 2. The façade structure consists of 63 compression struts, 96 tension rods and 96 square glass panels connected with rest of the structural members through four-way spider fittings as shown (Figure 1-b). The struts and the rods are made of S335 stainless steel and the glass panels from toughened glass. These materials have similar coefficients of thermal expansion, avoiding possible uneven length changes due to temperature change.
3. Analysis of the double-skin tensegrity glass façade

Analysis of façade wall was carried out in two steps. The first part includes the analysis of the inverse bidirectional tensegrity grids in order to determine some of their structural properties. The second part is the static analysis and preliminary design of the façade wall according to the load cases retrieved from the Eurocode.

3.1 Analysis of tensegrity grids

For the present study, analysis of tensegrity structures is carried out using a computational model for pin jointed frameworks, as formulated by [6], using the singular value decomposition (SVD) of the equilibrium matrix to determine the number of states of selfstress $s$ and the internal mechanisms $m$ of the tensegrity model, and their nature.

3.1.1 Analysis of the $2 \times 2$ module basic tensegrity grid

The tensegrity system considered for the analysis is an inverse bidirectional grid (Figure 1-a) augmented by planar diagonal struts which aim to replace the glass panels which form the top and bottom layers of the grid. Here we consider a $2 \times 2$ module grid used to form a large multi-modular tensegrity grid comprising 18 nodes and 53 elements. The augmented basic unit is composed of 12 tension rods (black lines) and 41 compression struts as shown in Figure 3.
The analysis carried out on the basic unit shows that this structure has five state of self-stress, $s=5$ and mechanisms $m=0$. This implies that the structure does not have any of the highly flexible modes of deformation often associated with tensegrity structures [7]. Among the 5 state of self-stress there is one that is fully symmetric which will stabilise the system. This stabilising self-stress state can be achieved through a pre-stressing pattern where all the vertical struts are lengthened as shown in Figure 4. In the figure all the tension elements are in tension and the diagonal bars aiming to replace the panels carry a low level of tension, but this is not critical to the design.

![Figure 4: A stabilising state of self-stress induced by the lengthening of the vertical compression struts. The elements undergoing tension and compression are coloured by yellow and red fills respectively.](image)

3.1.2 Analysis of the 8×6 module tensegrity grid
The 8×6 module grid is composed of twelve 2×2 module basic tensegrity grids. The structure is composed of 126 nodes and 489 elements of which 96 are tension elements. From the analysis it was observed that the structure possess $s=117$ and $m=0$ implying that the structure is rigid with a high degree of redundancy. In practice, a possible state of self-stress stabilising the system could be achieved by a pre-stressing pattern which includes the lengthening of all the vertical struts. Such a pre-stressing pattern provides stability by inducing tension in the tension rods, compression in the vertical and planar diagonal struts meaning that glass panels will undergo in-plane compressive forces.

3.2 Static analysis of the tensegrity glass façade wall
For the static analysis of the tensegrity glass façade, a linear elastic computational model was used, accounting the pre-stress of the system and external design loads. Compression steel struts and steel tension rods were used in the model with thin shells as glass panel sections. Additional FE analysis was performed on the glass panels due to their central role for the integrity and stability of the structure.

3.2.1 Pre-stressing
For the present computational model the pre-stress is applied through the lengthening of adjustable length compression struts. The amount of length extension applied was $e=5$mm, this value being found after a number of iterations considering three parameters: the amount of load applied on the structure, the allowable deflection and the fact that the 2.24m length tension rods should remain in tension (avoid slack tension rods or de-tensioning of the structure).

3.2.2 External loading & load cases
The loads considered for the tensegrity façade wall apart from the pre-stress are the wind loads (WL), the self-weight of the structure (SW) and temperature changes (T). The magnitudes of the
values and the allowable deflection criterion set for the present structure are listed in Table 1. The two load cases considered for the analysis were in accordance with the European code for loading of structures, Eurocode 1, and are listed in Table 2.

<table>
<thead>
<tr>
<th>Table 1: Applied loads.</th>
<th>Table 2: Load cases.</th>
</tr>
</thead>
<tbody>
<tr>
<td>WL</td>
<td>1.0 kPa</td>
</tr>
<tr>
<td>DL</td>
<td>1.07 kPa</td>
</tr>
<tr>
<td>T</td>
<td>±15°C</td>
</tr>
<tr>
<td>L/D</td>
<td>span/250</td>
</tr>
<tr>
<td>e</td>
<td>0.005 m</td>
</tr>
<tr>
<td>1.4•DL + 1.4•WL + Pre-stress</td>
<td>(1)</td>
</tr>
<tr>
<td>1.2•DL + 1.2•WL + 1.2•T + Pre-stress</td>
<td>(2)</td>
</tr>
</tbody>
</table>

### 3.3 Computational analysis and results

The computational model was analysed for the load cases shown in Table 2. From the analysis it was deduced that load case (1), was the most critical load case in terms of magnitude of loads applied on the members, whereas, load case (2) was the most critical for the analysis of the glass panels. Although the amount of external loads applied on the structure was extreme, the 5mm length extension applied on the compression strut was enough to maintain prestress and keep span deflection of the structure to minimum. The loads carried by the members are shown in Figure 5.

#### 3.3.1 Worst case scenario for struts and rods

In addition to the normal load cases, an extreme load case scenario was considered where, during the extreme load case (1) a glass panel is removed from the most critical area of the structure (Figure 5). In this case the loads carried by the members increased slightly comparing to the ones found when the glass panel was not removed. From the latter analysis results obtained for this case it can be deduced that the tensegrity glass façade has a degree of structural redundancy which prevents the exertion of high internal forces and significant deflections due to the loss of a glass panel validating the results of the analysis of the tensegrity model.

In this case the maximum forces carried by a tension rod are shown in Figure 6. The latter forces resisted by the members were below their maximum capacity shown in Table 4. The windward force applied on the structure caused a maximum deflection of 4.7mm (Figure 6) which is well
below the allowable deflection criterion shown in Table 1.

![Deformed shape of the façade wall](image)

**Figure 6: Deformed shape of the façade wall with maximum lateral deflection equal to 0.0047m.**

### 3.3.2 Worst case scenario for glass panels

From the analysis of the façade wall it was observed that the maximum load applied on the glass panels was during an extreme load scenario where the structure was loaded with load case (2) and the most loaded glass panel was removed. The glass panel undergoing the maximum load was the panel shown in Figure 5, which was subjected in excessive in-plane compressive forces in the Z and X directions and it is located next to the missing panel. The latter has a high slenderness and when subjected to high in-plane compression loads – such as the load case represented in Figure 7, it becomes important to investigate the possibility of glass failure by instability i.e. buckling.

A planar countersunk stainless steel bolted connection has been proposed since it allows direct load transfer through bearing of the glass on the bolt with a nylon 66 boss used as a liner material to avoid direct steel to glass contact. The bolt assembly is fixed onto a spider support system which connects four bolts from corners of four adjacent panels. A two layer sandwich laminated safety glass panel would be most ideal but in this analysis, a single layered glass has been considered for simplicity.

<table>
<thead>
<tr>
<th>Local joint no.</th>
<th>Fx (kN)</th>
<th>Fy (kN)</th>
<th>Fz (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>58.073</td>
<td>-1.155</td>
<td>-155.304</td>
</tr>
<tr>
<td>2</td>
<td>75.599</td>
<td>-1.263</td>
<td>150.348</td>
</tr>
<tr>
<td>3</td>
<td>-85.739</td>
<td>-1.134</td>
<td>126.095</td>
</tr>
<tr>
<td>4</td>
<td>-47.933</td>
<td>-1.249</td>
<td>-118.903</td>
</tr>
</tbody>
</table>

*Table 3: Maximum loads applied on the maximum loaded glass panel (force signs are associated with the axes direction shown in Figure 7)*
The 2 by 2m glass panel was analysed for bucking as a monolithic panel-like glass column under axial compression. The buckling analysis based on [8] shows that the critical buckling load of 223.5kN is not exceeded by the worst loading case, which has an axial compression of 134kN. The buckling stability check reveals that the tensile and compression stresses of 18.7MPa and 25.8MPa respectively, are well below the characteristic strength of thermally toughened glass, which is 120MPa [9].

The panel has also been numerically modelled as a single layered 19mm thick glass with four standard-sized bolt holes using thin shell QSL8 elements which are suitable for analysis of arbitrarily curved shell geometries with the element formulation taking account of both membrane and flexural deformations, the analysis was run using LUSAS with a total Lagrangian geometric nonlinearity option which is applicable for arbitrarily large deformation. The results of the FE analysis show that the general stress magnitudes on the surface of the panel are well below the characteristic strength of thermally toughened glass for the extreme load case shown in figure 7a below. However, there are some isolated stress peaks on the edges of the bolt holes and these are particularly high on the upper edge of hole number 3 (figure 7a) with a maximum principal stress of 214MPa. Although they are few and local, such stress peaks exceed the strength of glass and may result in failure in the vicinity of the bolt-hole. The problem of these high stress peaks can be resolved by making some modifications to the design of the tensegrity grid such as replacing bolted connections with linear adhesive connections on the panels to reduce stress peaks by allowing a more even stress distribution.

3.3.3 Sizing of structural elements

Tension and compression element sections were provided according to the results obtained from the worst case scenario. According to the member sections provided shown in Table 4, the weight of the actual structure was determined to be 107kg/m² and the weight of the structural façade enclosure approximately 12kg/m².

Table 4: Sections provided for the tensegrity glass façade.

<table>
<thead>
<tr>
<th></th>
<th>Max load (kN)</th>
<th>Section assigned</th>
<th>Section capacity (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Compression strut</strong></td>
<td>165.8</td>
<td>M36 CHS (mm)</td>
<td>186</td>
</tr>
<tr>
<td></td>
<td></td>
<td>od: 88.9</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>t: 5</td>
<td></td>
</tr>
<tr>
<td><strong>Tension bar</strong></td>
<td>271.1</td>
<td>M36 Bar (mm)</td>
<td>376</td>
</tr>
<tr>
<td></td>
<td></td>
<td>dia: 34</td>
<td></td>
</tr>
<tr>
<td><strong>Glass panel</strong></td>
<td>(see section 3.3.2)</td>
<td>Glass panel (mm)</td>
<td>(see section 3.3.2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>lxd: 2000×2000</td>
<td>t = 19</td>
</tr>
</tbody>
</table>
4 Conclusions

The results from the analysis of the proposed system showed that the tensegrity system is capable of resisting the design forces with lightweight sections undergoing negligible deflections. The pre-stressing pattern used was able to provide the amount of pre-stress required avoiding possible detensioning and significant deflections of the structure even in the extreme load cases. The analytical investigation of the glass panel under extreme loading case has shown that the panel will not fail by bucking. FE analysis showed that the global surface stress magnitudes were within permissible stress for glass, however, there are some localised high stress peaks on the edges of the bolt-holes which could be avoided using linear glass connections.

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6 References


