A prediction tool for the optimisation of maintenance activity to reduce disturbance due to ground-borne vibration from underground railways

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Abstract

Deterioration in the track and rolling stock of underground railways can lead to increased noise and vibration levels in buildings, which can then cause disturbance to their occupants. One aim of Work Package 8 of the EC-Growth project CONVURT (CONtrol of Vibrations from Underground Railway Traffic) is to develop ways of monitoring underground railways such that maintenance activity may be optimised to reduce disturbance due to ground-borne vibration.

A convenient means of identifying deterioration in the track is by measuring the resultant increase in vibration levels on the axle-boxes of trains. Once deterioration has been identified, it is desirable that maintenance programmes are then prioritised in order to remedy those sections of track that are most at risk of causing disturbance in buildings.

This paper describes the prediction tool developed as part of Work Package 8 to aid this process. By taking measured changes in axle-box vibration, accounting for the essential details of the rolling stock, transmission path and receiver, the resultant changes in surface vibration levels may be predicted. The paper includes an overview of the underlying vibration propagation model, together with an example application based on measured data.

1. Introduction

Railway track is not perfectly smooth and, with time, its surface roughness increases. Along with other forms of deterioration, such as settlement at rail joints, this leads to an increase in the vibration generated as the wheels of a train roll along the track. From the wheel-rail interface, the vibration may propagate through the track and tunnel structure, through the ground and into buildings, where it may then cause disturbance as perceptible vibration and/or re-radiated noise. Typical vibration levels lie in the range from 0.1 to 1.0 mm/s and peak in the frequency range from 40 to 80 Hz [1, 2]. Although these levels are unlikely to cause even light damage to buildings, the disturbance caused to building occupants and the disruption caused in specialist buildings, such as hospitals and research facilities, can have significant social and economic consequences [3].

In order to control this problem effectively, underground railway operators must quantify the level of deterioration of the track on a regular basis and across each rail network. A new method of doing this has been developed by three other CONVURT partners, London Underground Ltd, RATP (Paris) and ATM (Milan). The technique involves using accelerometers to record the vibration levels on the axle-boxes of trains as they move around each network. Axle-boxes are the

box-like structures that support each axle on a bogie. By mounting accelerometers at each end of an axle-box, directly above each wheel, measurements are made as close as possible to the wheel-rail interface. Vibration in the vertical direction is measured as this is the most representative of the rail head condition. Every six months, an instrumented service train is sent down each railway line to record the vibration levels, along with simultaneous measurements of the train's speed and location. The level of deterioration in the track is then quantified in terms of the increase in axle-box vibration, over that six-month period, as a function of distance along each rail.

The measurement of axle-box vibration provides a practical, cost effective means of identifying deteriorating sections of track. So that maintenance activity can be prioritised, it is also desirable to identify those sections of track that are most at risk of causing disturbance in nearby buildings. This second step requires a systematic means of determining the effect of rail deterioration on the vibration levels experienced on the surface of the ground.

The prediction tool described here has been developed to aid this process. It involves two stages: firstly the axle-box acceleration data are processed to characterise the track in terms of the forces generated as a train runs along it; secondly these forces are applied to a propagation model that predicts the resulting vibration levels on the ground.

2. Characterising the track

To predict the surface vibration levels due to trains running on a particular track, it is first necessary to characterise that track in terms of the wheel-rail interface forces generated as a train runs along it. The method described here begins with the assumption that two continuous acceleration time-series have been measured, one for each rail, and that further data are available to locate the position of the train on the track at any point in time. Only roughness-generated vibration is considered – impulsive events, such as dipped joints are not treated explicitly.

The time series may have been measured by just two accelerometers, one at either end of the same axle-box, or they may have been derived from measurements made over several axles. In either case, the following primary assumptions are made concerning the vehicles and track. In practice, railway operators may wish to relax some of these assumptions if more comprehensive models or measurements are available. The assumptions made here are considered reasonable given the complexity of the propagation model described in Section 3.

1) Each time series is representative, given suitable time shifts, of the axle-box accelerations experienced by all wheels running on the particular rail. This assumes the characteristics of the vehicles, bogies and wheels do not vary from axle to axle. The latter includes the assumption of a uniform level of wheel roughness.

2) It is assumed that any wheel-rail interface force may be calculated by multiplying the relevant axle-box acceleration by a nominal value representing the axle's un-sprung mass. The value used here is the average un-sprung mass of the train. Details of the vehicle dynamics are ignored and, in particular, wheel-wheel coupling along and between axles is neglected.

3) Any rail-rail coupling, such as torsional coupling via the track bed, is ignored.

4) Any extraneous noise in the recorded data, such as interference from drive motors, track signalling, etc., is neglected.

The data processing consists of 2 main steps: (1) calculation of the axle-box acceleration spectral density functions, or *spectra*, for each rail; (2) calculation of the corresponding spectra for the wheel-rail interface forces. These steps are considered in detail below.

2.1. Calculation of axle-box acceleration spectra

The propagation model requires data describing both the spatial variation in vibration levels along each rail and the variation in levels with frequency. Here the objective is to derive a set of acceleration spectra, where each spectrum represents the frequency response of the axle-box as it runs over a particular section of rail. An essential aspect of the data processing is the necessary compromise between the accuracy and frequency resolution of the derived spectra, and the spatial resolution along the rail.

Consider data acquired by a train running at speed v. The method described in this section applies equally to data describing either rail; the results from both rails are combined in Section 2.2. A sampling frequency of $f_s = 500$ Hz is used in order to accommodate a frequency range up to 250 Hz – the typical range of ground-borne vibration – while minimising data storage requirements.

Spatially, the rail is divided into sections of equal length $L_{\sec tion}$. The length T_{sample} of the time samples acquired over each rail section vary in length according to the speed of the train:

$$T_{sample} = \frac{L_{\sec tion}}{v} \tag{1}$$

Accuracy and frequency resolution improve with the length of the time sample but this is at the expense of reduced spatial resolution in terms of an increase in the required rail section length. For each time sample, the frequency resolution and statistical accuracy of the derived spectrum are related and depend on the method of averaging [4, 5]. The method adopted here involves partitioning the time sample into K overlapping segments. Each segment is then Fourier transformed separately and the resulting K periodograms averaged at each frequency. The optimum amount of overlap is equal to one half of the segment length, giving a spectral accuracy of:

$$\frac{\sigma}{m} \approx \sqrt{\frac{11}{9K}} \tag{2}$$

where σ is the standard deviation and m the mean of each spectral estimate, and K is the number of overlapping segments. The appropriate number of segments is determined by considering the required frequency resolution. A value of $K = 11$ gives an acceptable accuracy of $\sigma/m = 0.33$, which is further improved if lower resolution measures, such as a one-third octave band spectrum or r.m.s. level, are used as the final output.

The maximum train speed encountered in practice is approximately 20 m/s. With a rail section length of 10 m, equation (1) gives $T_{sample} = 0.5$ s. Given that $K = 11$ corresponds to 6 *independent* data segments, the final frequency resolution is:

$$\Delta f = \frac{1}{T_{sample}/6} = 12 \text{ Hz} \tag{3}$$

This resolution is considered to be an acceptable compromise for the form of output envisaged. The vibration levels associated with underground railways typically peak in the one-third octave bands centred on either 63 or 80 Hz [2], which have bandwidths of approximately 15 and 18 Hz respectively. A frequency resolution of 12 Hz therefore provides reliable one-third octave band spectra over this frequency range, albeit with some error in the narrower bands at lower frequencies. Of course, many trains run slower than 20 m/s and, in these cases, the longer time samples will provide improved frequency resolution.

One further consideration is the problem of spectral leakage [4]. To reduce this, a Hanning window is applied to each data segment prior to applying the Fourier transform. Since this window falls to zero at its ends, an extra data segment must be added to the time sample. This segment extends into the following time sample to ensure that no data are lost at the sample boundaries, as illustrated in Fig. 1.
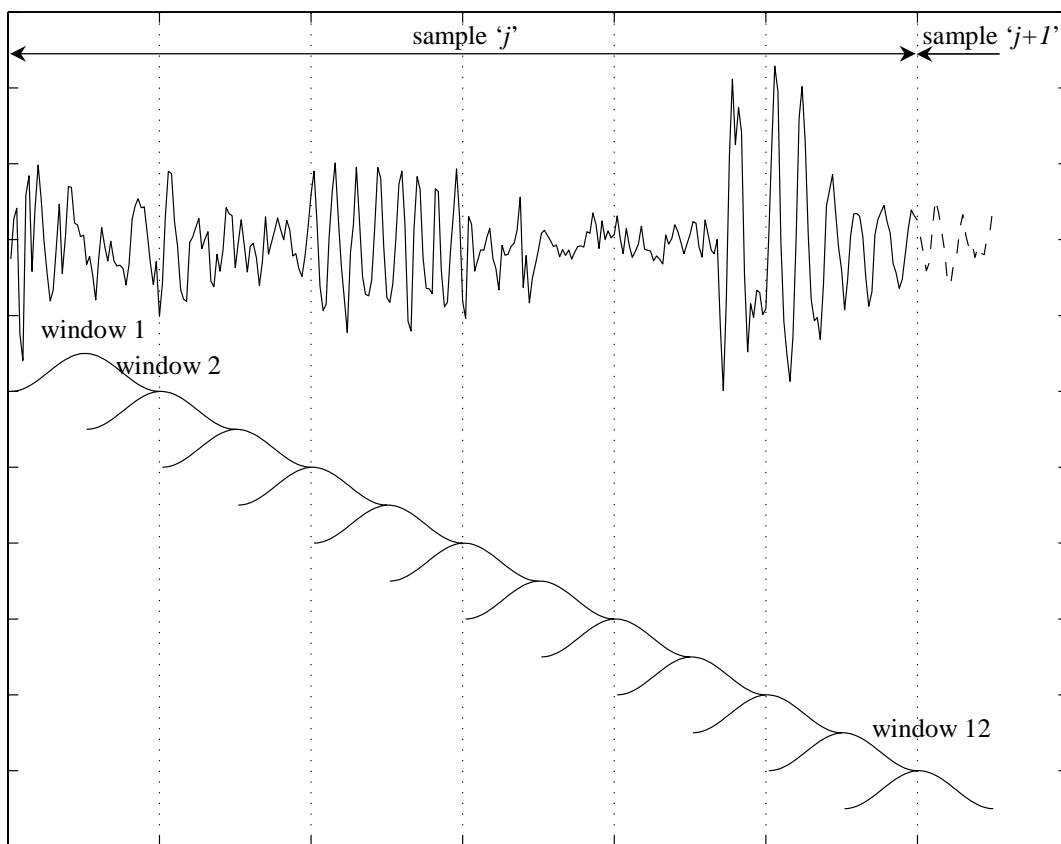

Fig. 1. A summary of the data processing routine applied to each axle-box acceleration time-sample to generate the corresponding spectrum. The case of 11 overlapping segments is illustrated. Note the extra twelfth segment extending into the following time sample to ensure no loss of data at the sample boundaries.

In summary, provided with an acceleration time-series describing a particular rail, sampled at a frequency of at least 500 Hz, the following processing routine is used.

1) Low-pass filter with a cut-off frequency of 500 Hz.

2) Divide into time samples corresponding to consecutive 10 m lengths of rail – the number of data N_{sample} in each will vary according to the speed of the train.

3) For each time sample:

   i. divide into 11 overlapping segments with 50% overlap, each segment containing $N_{sample}/6$ data;

   ii. extend the time sample with a further $N_{sample}/12$ data from the following sample – in order to accommodate a twelfth segment;

   iii. apply a Hanning window to each segment, zero-pad to the next highest power of two and Fourier transform;

   iv. average together the resulting 12 periodograms to produce the final spectrum.

2.2. Calculation of wheel-rail interface force spectra

Given two acceleration time-series, one for each rail of a particular track, the process described above produces two sets of acceleration spectra $S_{a_l}^j$ and $S_{a_r}^j$, where the 'j^{th}' pair of spectra describe the frequency response of a wheel as it runs over the 'j^{th}' section of the left and right rails. Following the assumptions stated earlier, the spectra for the wheel-rail forces are obtained by multiplying the acceleration spectra by the average un-sprung mass of the train m:

$$S_{f_l}^j = m^2 S_{a_l}^j \quad \text{and} \quad S_{f_r}^j = m^2 S_{a_r}^j \tag{4}$$

The propagation model described in Section 3 uses a simplified representation of the track. Torsion of the track bed is ignored and the two rails are treated together as a single beam. Over a given rail section, and given the assumptions stated earlier, the force signals may be treated as random uncorrelated signals. The two sets of spectra given by equation (4) may therefore be combined into one set for application to the rail beam as follows [4]:

$$S_f^j = S_{f_l}^j + S_{f_r}^j + 2S_{f_l f_r}^j \tag{5}$$

where $S_{f_l f_r}^j$ is the cross-spectrum between the force signals for the 'j^{th}' section of the left and right rails.

The entire processing routine is illustrated in Fig. 2.

axle-box acceleration time-series (one for each rail)

acceleration time-samples

sample 'j'

data processing

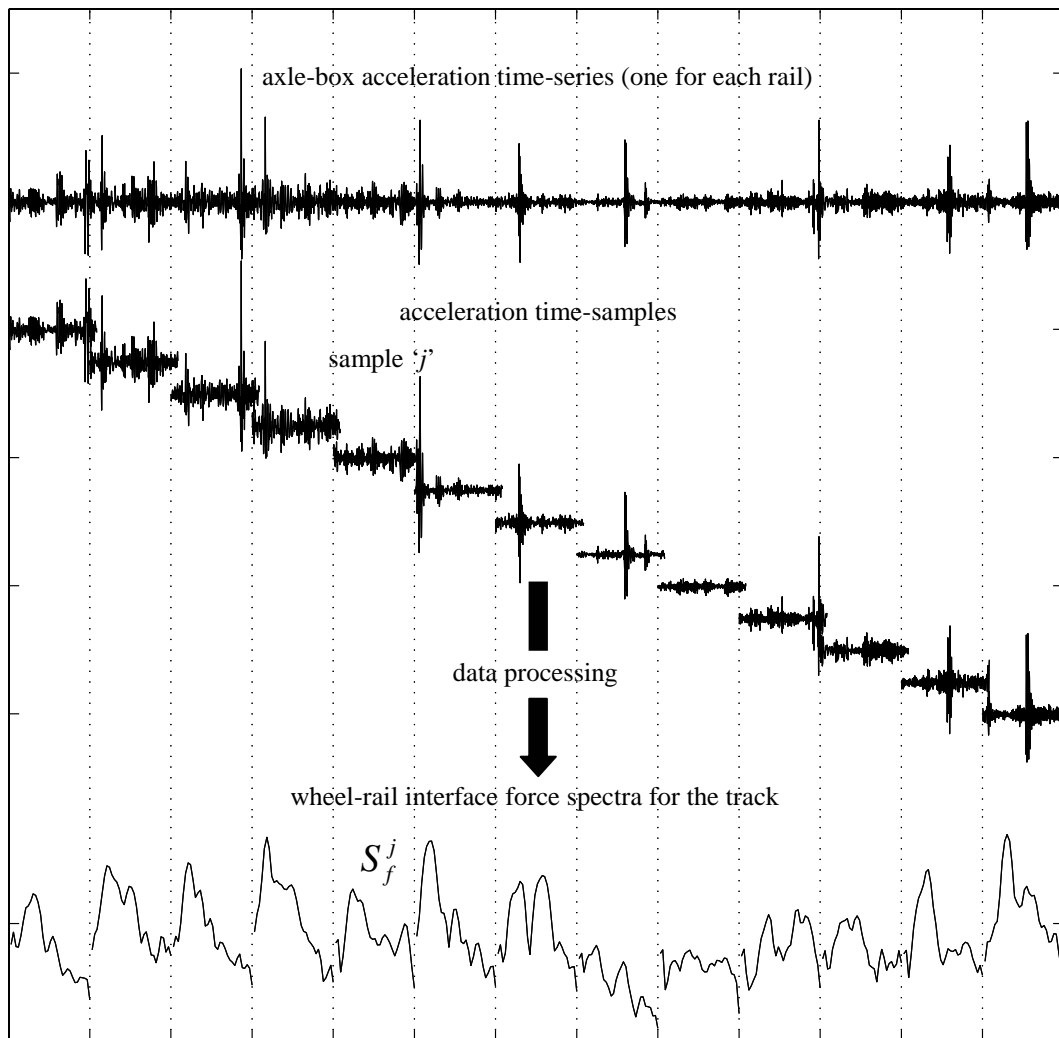wheel-rail interface force spectra for the track

S_f^j

Fig. 2. A summary of the data processing routine applied to each axle-box acceleration time-series to generate the corresponding set of force spectra. The case of a 130 m length of track is illustrated, with each spectrum representing the wheel-rail interface forces generated as the train runs over consecutive 10 m sections of track.

3. The propagation model

The prediction of absolute levels of noise and vibration in buildings due to underground railways is a specialist task requiring complex theoretical models and a detailed list of input data. Current techniques involve coupled finite-element/boundary-element models, and the required input data include rail roughness profiles and the dynamic characteristics of the ground, as well as detailed information on the tunnel and building structures [6, 7]. While this approach is valuable in special cases, it is not a practical option for frequent use by railway operators.

The propagation model required for the work described here must account for the essential details of the track, tunnel and ground such that *changes* in surface vibration levels may be predicted given the measured deterioration of the track. A key requirement is that the model is computationally efficient and suitable for use on a standard personal computer, so that rational decisions about track maintenance can be made quickly and without the need for extensive site investigations or structural detail. This section describes one approach that is suitable for use with

deep bored tunnels. The overall approach is generally applicable but it is envisaged that, in practice, railway operators will select models according to their specific needs.

3.1. An approach for deep bored tunnels

Once the spectra of the wheel-rail interface forces have been determined according to Section 2, vibration levels on the free surface of the ground are calculated in two stages, corresponding to the near-field behaviour of the tunnel and the subsequent vibration propagation to the free surface. Further details of the operation of the model are given in Section 4; this section presents an overview of the model components.

3.1.1. Stage 1: near-field behaviour of the tunnel

The tunnel model accounts for the essential three-dimensional dynamic behaviour of the track, tunnel and soil of a deep bored tunnel. The model is based on the analytical approach of Forrest and Hussein [8, 9], which treats the track and tunnel as infinitely long. This enables closed-form solutions to be used to ensure computational efficiency. Fig. 3 illustrates the essential components.
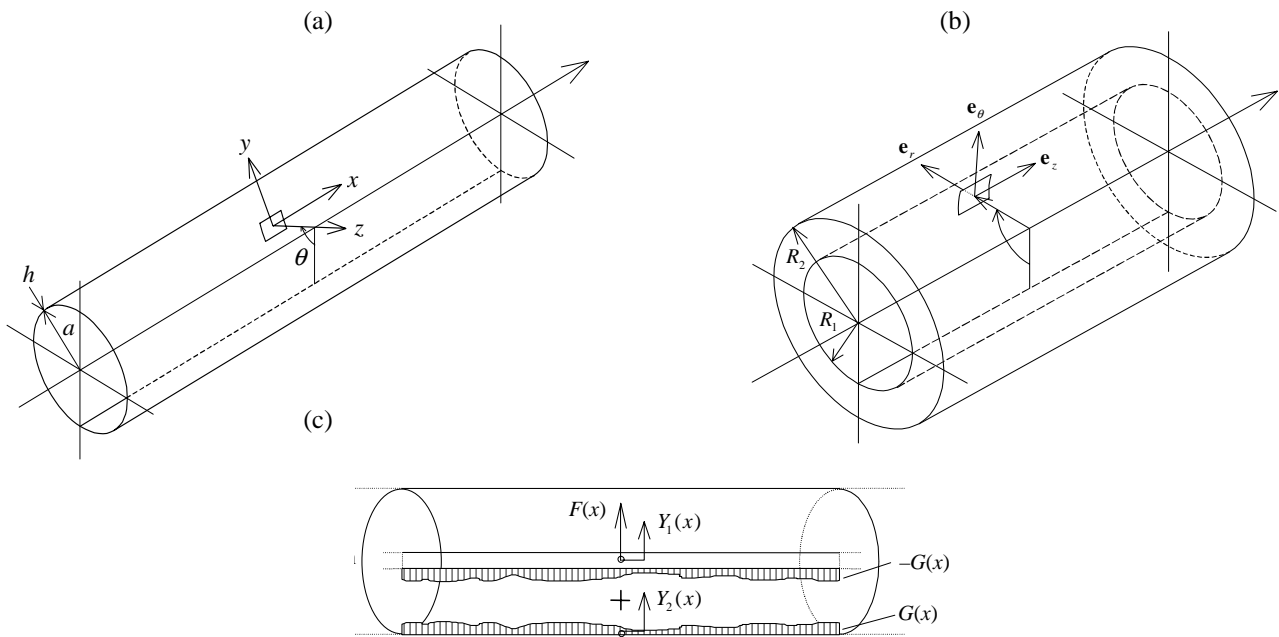

Fig. 3. The tunnel model consists of (a) a thin-walled cylindrical shell representing the tunnel, coupled to (b) a thick-walled cylinder representing the surrounding soil, whose inner radius equals that of the tunnel and whose outer radius is made infinite. Both are assumed infinitely long and are coupled to a track model (c) comprising Euler beams.

The tunnel itself is modelled as a thin-walled cylindrical shell. The surrounding soil, which is included to ensure representative near-field behaviour of the tunnel, is modelled as a three-dimensional elastic solid. This takes the form of a thick-walled cylinder whose inner radius equals that of the tunnel and whose outer radius is made infinite. The analytical solutions for these two components are coupled together using appropriate stress and displacement boundary conditions.

The track model comprises two Euler beams, one to represent the combined bending behaviour of the rails and one to represent the track bed. These are coupled to the tunnel invert via a convolution integral.

The principal function of the tunnel model is to generate the soil-tunnel interface stresses due to a unit point force applied in the vertical direction to the rail beam. Stage 2 then applies these stresses to a half-space vibration propagation model, which calculates the resulting displacements at the required locations on the free surface.

3.1.2. Stage 2: vibration propagation to the free surface

The half-space propagation model is described by the analytical Green functions developed by Tadeu *et al*. [10]. These describe the steady-state response of a homogeneous three-dimensional half-space to a buried time-harmonic line load whose amplitude varies sinusoidally along a line lying parallel to the free surface. They are sometimes referred to as two-and-a-half dimensional Green functions and may be used to calculate the time-harmonic response to any spatially varying line load by decomposing the latter into its wavenumber components. The functions are employed here by assuming that the free surface of the ground is sufficiently above the tunnel for the latter to behave as a line load, as illustrated in Fig. 4.
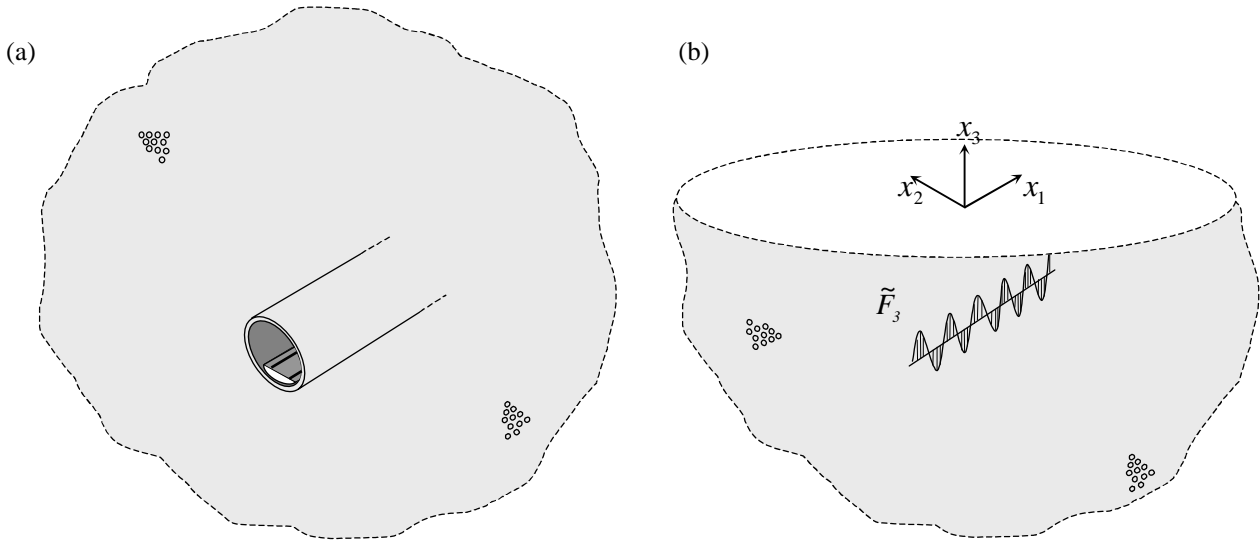

Fig. 4. Free-surface vibration levels are calculated by reducing the soil-tunnel interface stresses, calculated from the tunnel model (a), to an equivalent line load which is then applied to the half-space vibration propagation model (b).

The line load comprises three orthogonal components that are obtained by integrating the soil-tunnel interface stresses obtained from Stage 1 around the circumference of the tunnel. These are then applied to the half-space at the location where the tunnel centroid would lie if it was modelled explicitly. The resulting displacements are calculated using the Green functions describing propagation from the location of the tunnel centroid to a specified location on the free-surface.

In this way, the frequency-response functions (FRFs), describing the free-surface responses to a unit point force on the rail beam, may be calculated. Given the infinite length of the tunnel, the input force and observation point on the surface can be shifted longitudinally, while maintaining their separation, and the response at the observation point will not change. Hence, the problem of finding the set of FRFs for the surface response at $x_1 = 0$ due to a set of forces at various positions along the rail beam can be recast into the problem of finding the FRFs for the surface at those same positions to a single force at $x_1 = 0$, as illustrated in Fig. 5. The total soil response due to the full set of forces acting simultaneously is calculated by combining all the FRFs for the force at $x_1 = 0$, after scaling and phasing each one appropriately. The latter is achieved using random process

theory [4]. For a train with N_a axles located at a known location on the track, the spectrum of the displacement u at a particular location on the free-surface, due to the known set of wheel-rail interface forces f_k, is given by:

$$S_u = \sum_{p=1}^{N_a}\sum_{q=1}^{N_a} H_p^* H_q S_{f_p f_q} \qquad (6)$$

where H_p and H_q are the FRFs of u due to the forces f_p and f_q respectively (* denoting complex conjugate), and $S_{f_p f_q}$ is the cross-spectrum between these forces. As stated in Section 2.2, the force signals are treated as uncorrelated, and therefore:

$$S_{f_p f_q} = 0, \qquad \text{for } p \neq q \qquad (7)$$
$$S_{f_p f_q} = S_f^k, \qquad \text{for } p = q = k$$

where S_f^k is the spectrum of the wheel-rail force for the 'k^{th}' axle, which is selected from the set of spectra S_f^j, calculated in Section 2, according to on which track section the axle is located.
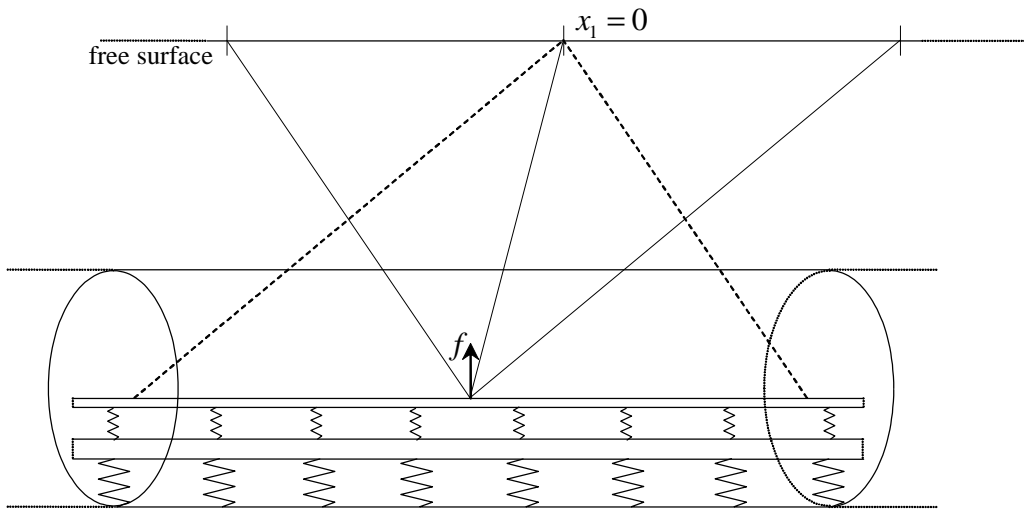

Fig. 5. Given the infinite length of the tunnel model, the response at one point on the free surface due to each wheel force (paths with dashed lines) is obtained by combining the separate responses to the single input shown (paths with solid lines).

Once the spectrum of the free-surface displacement has been calculated, this may be converted into the required final output, such as an equivalent one-third octave band spectrum or r.m.s. level. The calculation may then be repeated for different locations of the train and for any location on the free surface.

4. Operation of the prediction tool

This section summarises the operation of the prediction tool by considering the example of calculating the change in the peak r.m.s. displacements, over a six month period, experienced at locations on the surface of the ground directly above a tunnel. This is illustrated with the help of data acquired during a series of measurements made on London Underground's Bakerloo Line [2].

The basic data set comprises two pairs of acceleration time-series, each pair comprising data describing the left and right rail, measured six months apart by the same service train in the same overall condition. Further data locate the speed and position of the train at any point in time. In this example, the speed of the train is constant across the 200 m length of track. The process outlined in Section 2 is used to produce two sets of wheel-rail force spectra representing the state of the track six months apart, which are then used as inputs to the propagation model. The propagation model is tailored to the particular situation using the best available parameters to describe the track, tunnel and soil.

Fig. 6 shows the r.m.s. vertical displacement, as a function of distance along the free surface, for three locations of the train. In this example, the rail roughness is similar along the entire track. If the calculation is repeated by moving the train along the track in 10 m increments (equivalent to the spatial resolution of the force spectra), the peak r.m.s. levels may be calculated as the envelope of the individual r.m.s. curves. Note that, although the predictions are labelled as r.m.s displacement, these should not be regarded as absolute values. The methods described here are only regarded as being sufficiently comprehensive for calculating *changes* in levels.
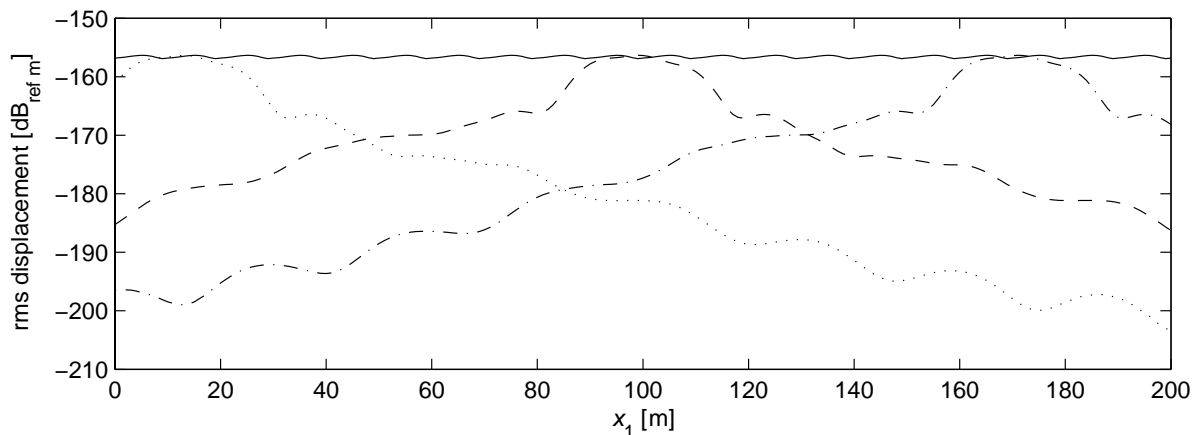

Fig. 6. The r.m.s. vertical displacement, over the frequency range from 0 to 250 Hz, as a function of distance along the free surface directly above the tunnel. The individual curves for three locations of the train are shown, along with the final result in terms of the peak r.m.s. level experienced as the train moves along the entire track (solid line).

By repeating the calculation with the axle-box data acquired six months later, any changes in the track condition may be assessed. Fig. 7 simulates the effect of a doubling of the rail roughness over the central 10 m section of track. The resulting increase in free-surface vibration is clearly visible. In practice, the changes in roughness level will vary along the track, as well as the nature of the land use on the surface. Some areas of land may be potentially sensitive, such as residential areas, while others may not, such as open parkland. By combining this land-use data with the output from the prediction tool, decisions may be made regarding which sections track require high-priority maintenance.
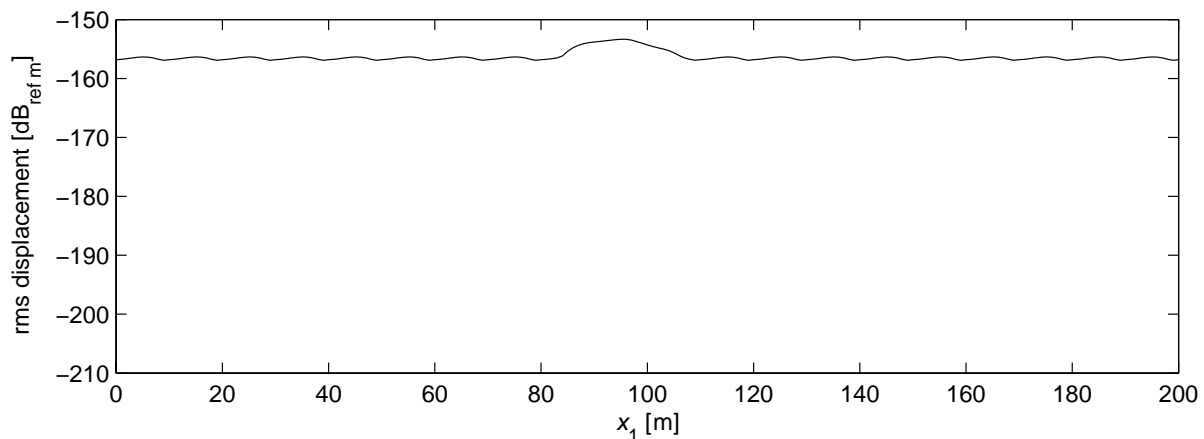
Fig. 7. The r.m.s. vertical displacement, over the frequency range from 0 to 250 Hz, as a function of distance along the free surface directly above the tunnel. The effect of a doubling of the rail roughness over the central 10 m section of track is simulated.

5. Conclusion

A key concern of underground railway operators is the disturbance caused by ground-borne vibration generated as a result of deteriorating track. The measurement of axle-box vibration provides a practical, cost effective means of identifying this deterioration. However, it does not identify those sections of track that are most at risk of causing disturbance on the surface of the ground. This paper has described a prediction tool that has been developed to aid this second step, thereby enabling maintenance activity to be prioritised.

Only an overview of the prediction tool has been presented. The practical application of the tool requires consideration to be given to details such as track curvature, varying train speeds and impulsive events, rather than just rail roughness. These are the subject of ongoing work.

Acknowledgements

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